
A Brief Refreshing Course of Computer Graphics

Tobias Isenberg

The Overall Goal: Photorealism



Gilles Tran, using POV-Ray 3.6

The Second Goal: Speed and Efficiency



Doom (1993), by id Software

The Second Goal: Speed and Efficiency



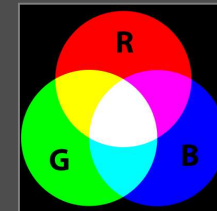
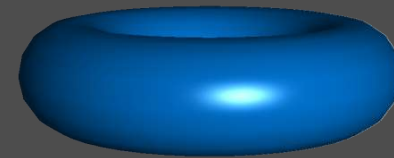
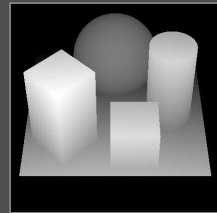
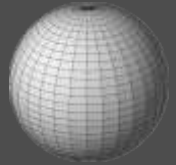
Ryse (2013); by Crytek/Microsoft Studios

Computer Graphics 101

- computer graphics pipeline
- models and object representation
- transformations and projections
- hidden surface removal
- illumination and shading
- texture mapping
- color and color models



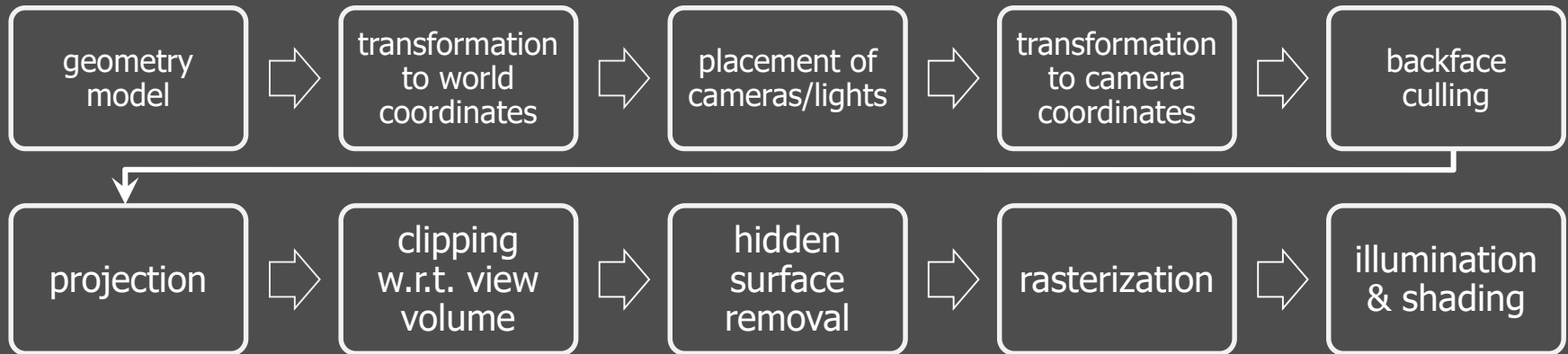
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Computer Graphics Pipeline

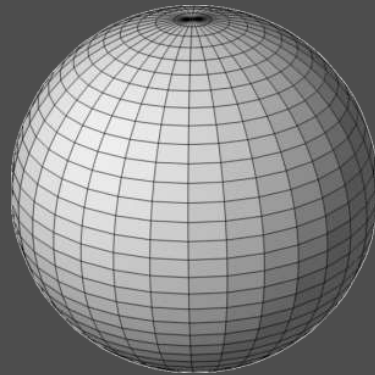


Computer Graphics/Rendering Pipeline



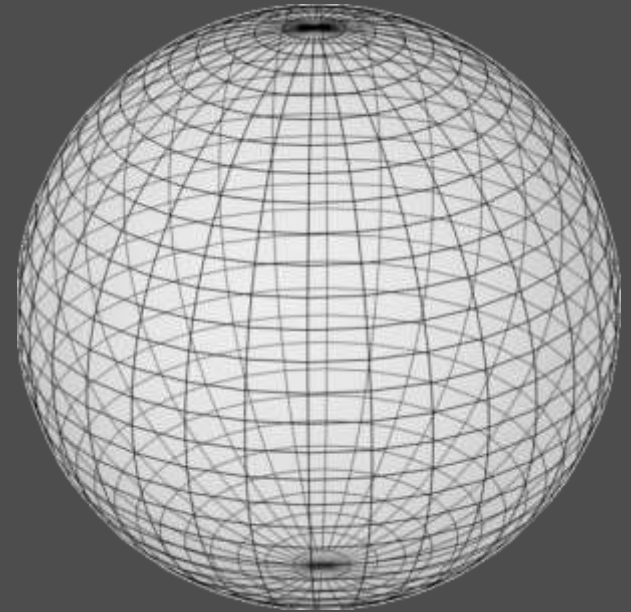
- simple model of physical processes
- independent parallel processing of triangles
- control/parameterization of the individual stages
- implementation of the specific stages in hardware

Models and Object Representation



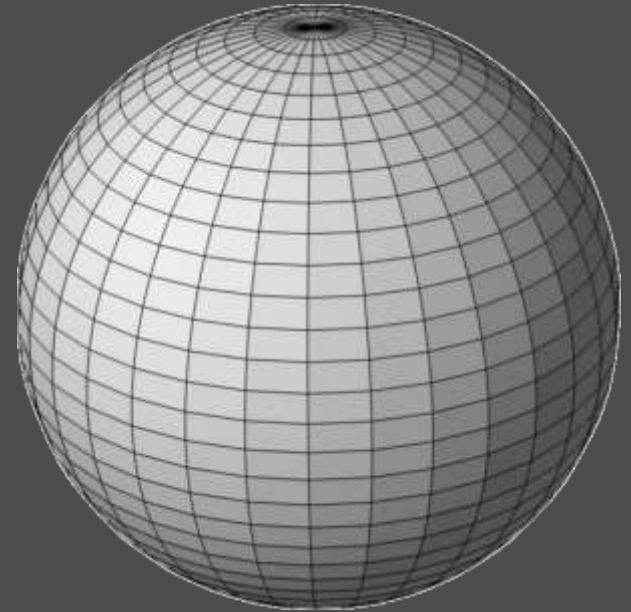
What to Specify for a 3D Shape?

- geometry
 - shapes, positions
 - connectivity, inside/outside
- material properties
 - visuals, textures
(plastic, wood, metal, etc.)
 - other material properties
(elasticity, mass, etc.)
- behaviour/animation
- more depending on the specific application



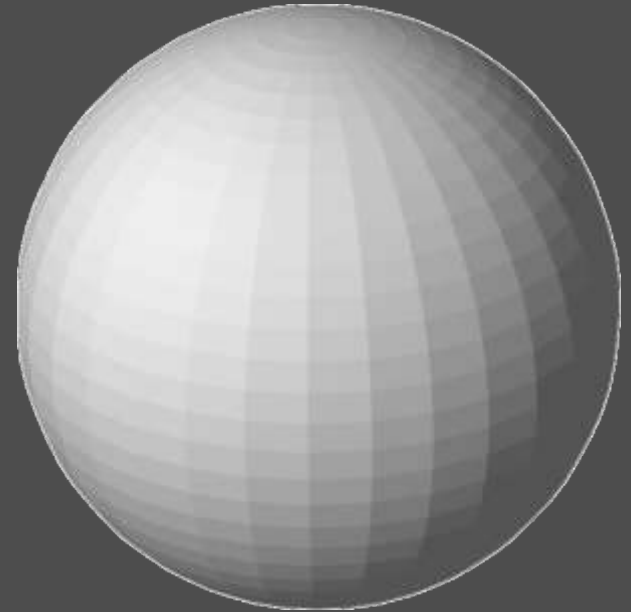
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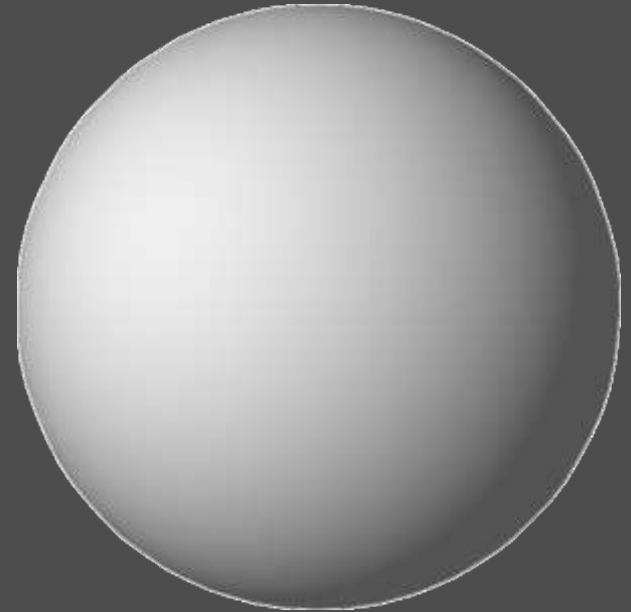
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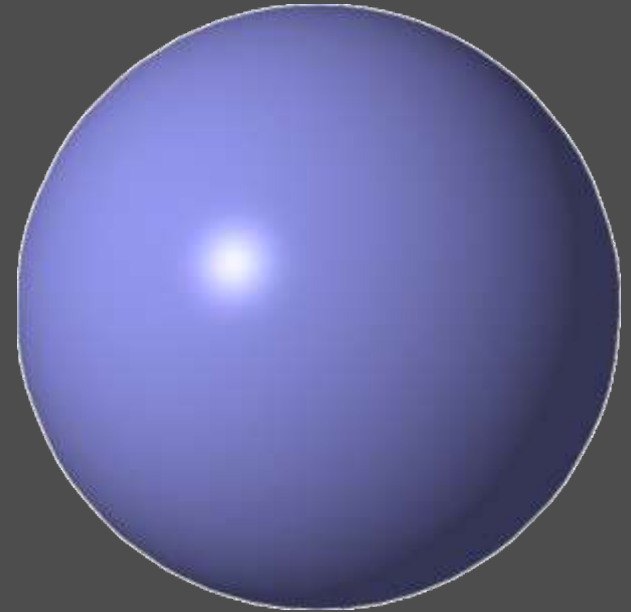
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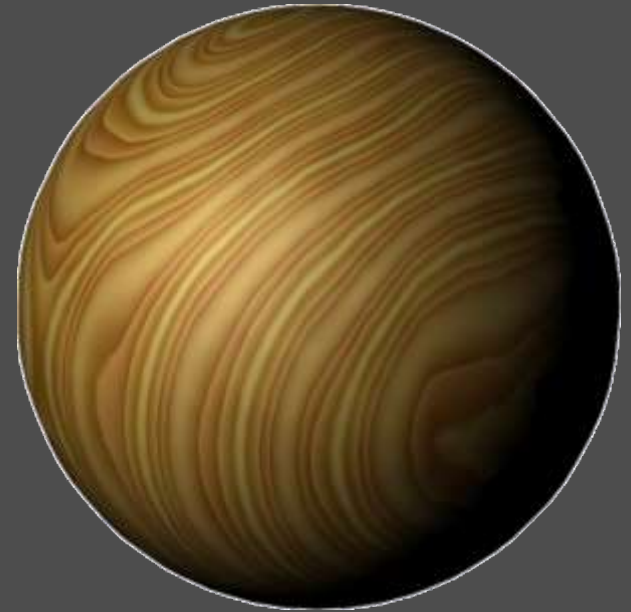
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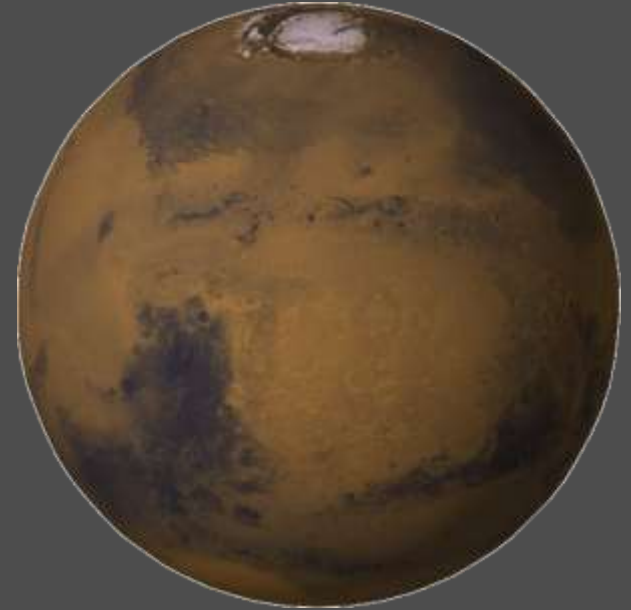
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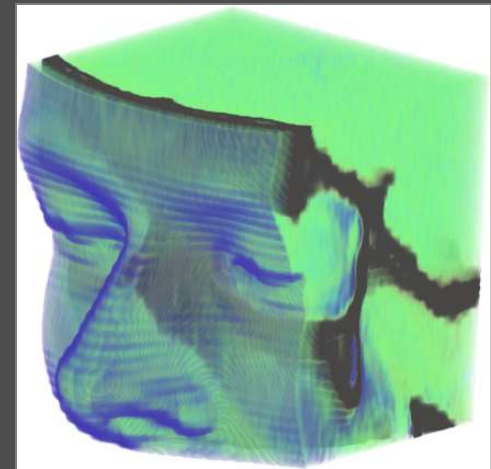
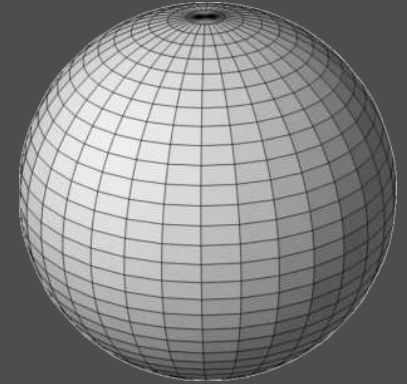
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How to Specify a 3D Geometry?

- boundary representations (b-reps)
 - meshes
 - piecewise smooth surfaces
- volume representations
 - voxel models
 - implicit surfaces
 - CSG: constructive solid geometry
 - space partitioning
 - BSP trees: binary space partitioning
 - octrees



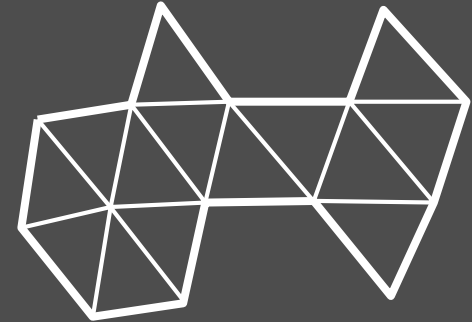
B-Reps: Polygonal Meshes

- polygons to define the surface of objects



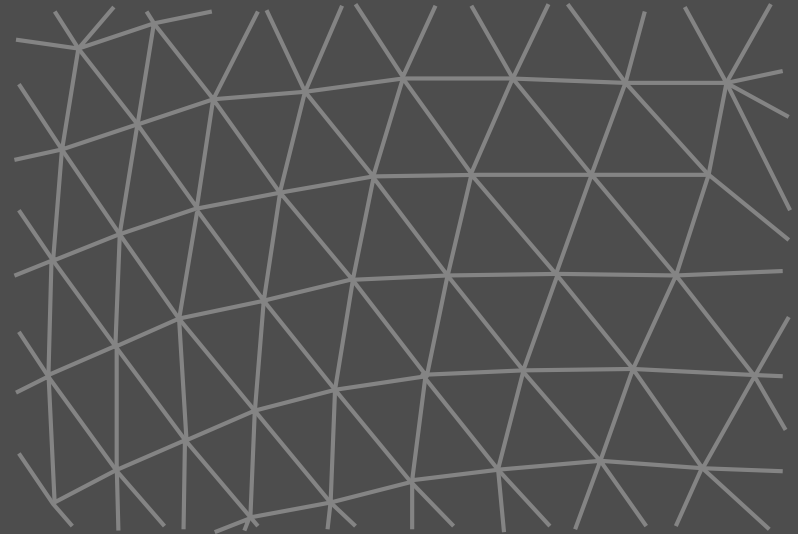
B-Reps: Polygonal Meshes

- polygons to define the surface of objects
- triangle meshes
 - polygon with fewest vertices
 - always convex & planar
 - defines unique surface



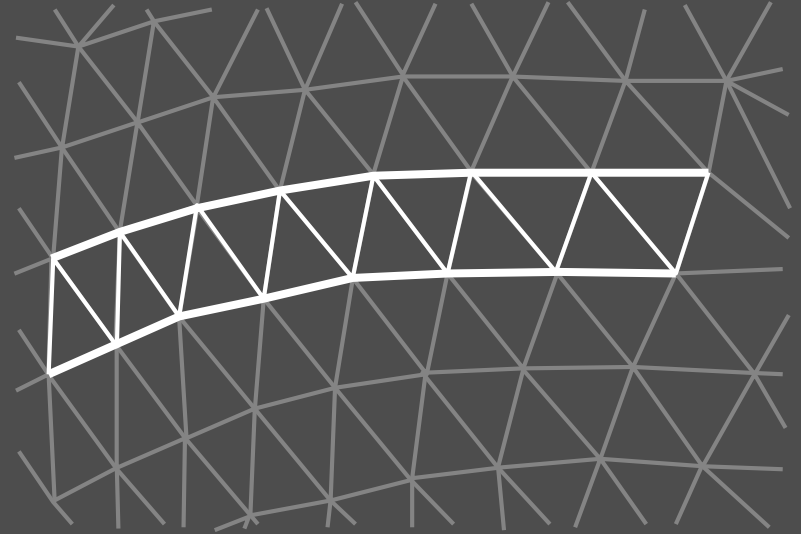
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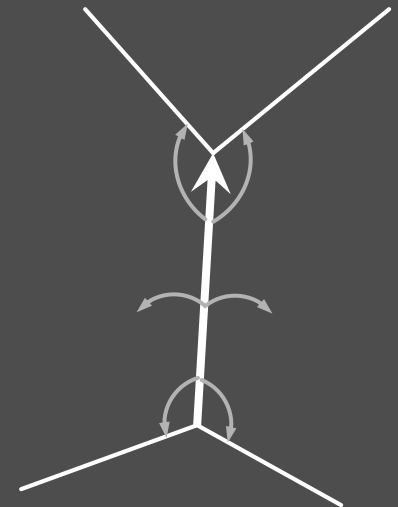
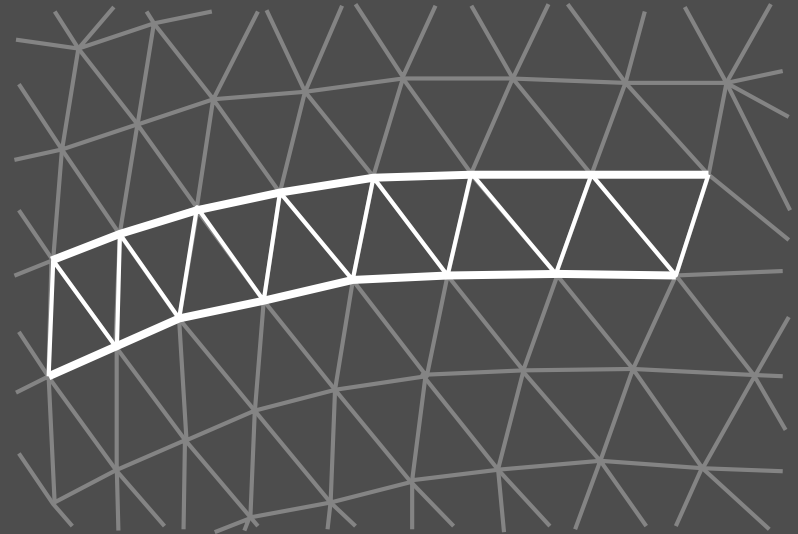
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→ defines unique surface
- triangle strips: faster rendering



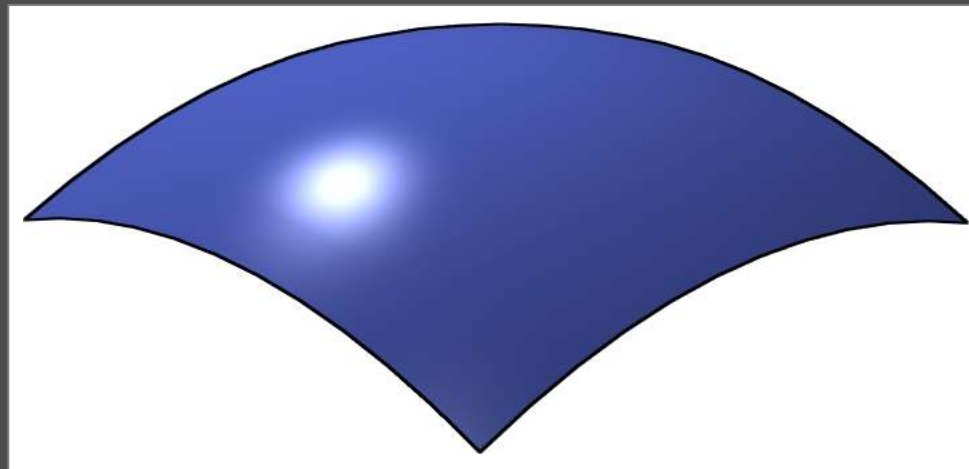
B-Reps: Polygonal Meshes

- polygons to define the surface of objects
- triangle meshes
 - polygon with fewest vertices
 - always convex & planar
→ defines unique surface
- triangle strips: faster rendering
- more complex mesh data structures (e.g., Winged Edge)



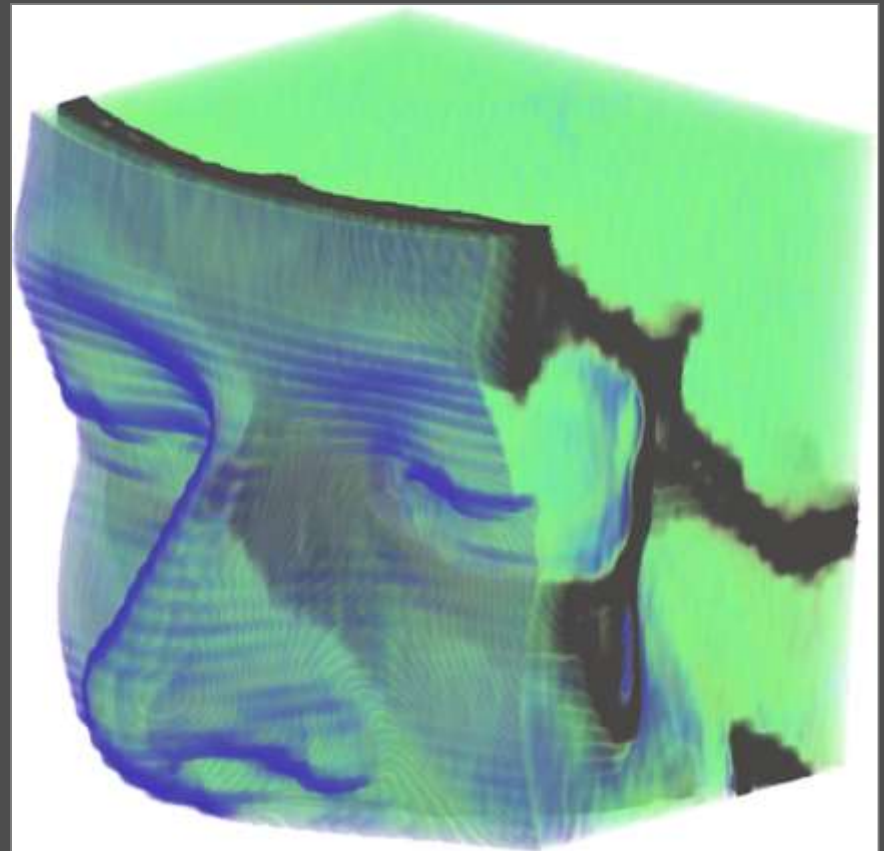
B-Reps: Piecewise Smooth Surfaces

- surface constructed from patches
- patches can be curved and are smooth
- patches satisfy a continuity constraint
- e.g., Bézier, Spline, NURBS surfaces



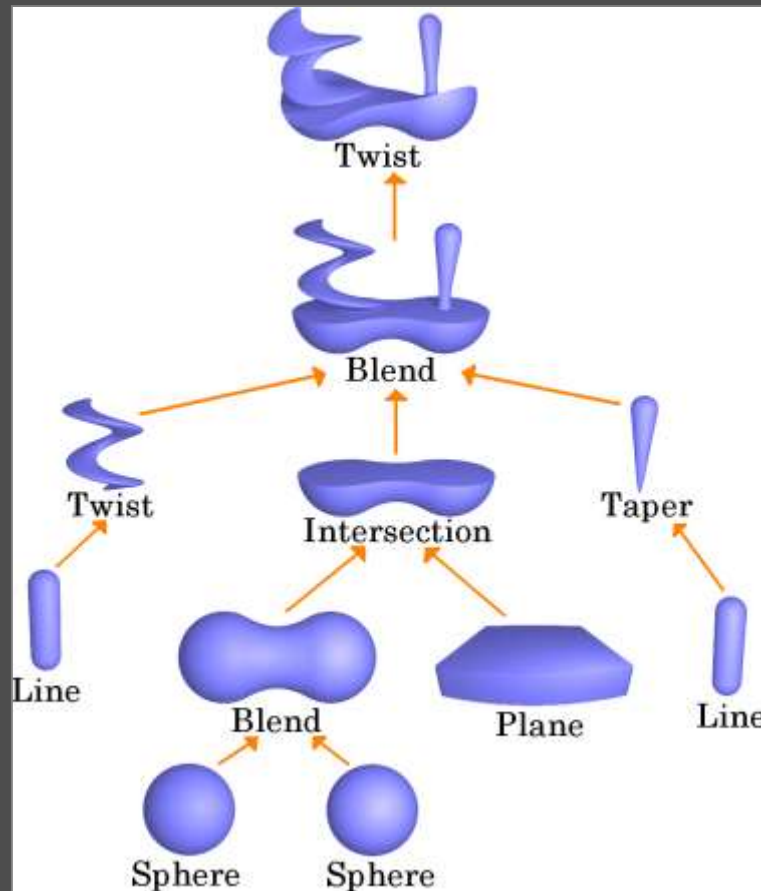
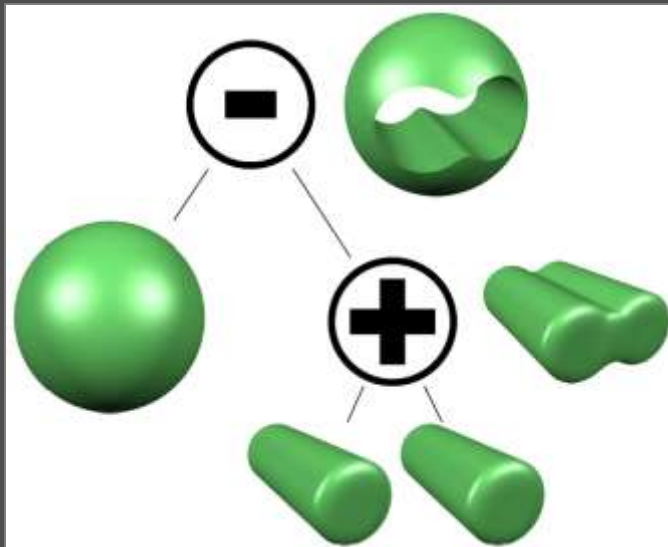
Volumes: Voxel Models

- sampling of a volume in regular intervals
- samples as cubes, or as general boxes
- several properties can be sampled
- shapes: *iso-values* define *iso-surfaces*
- heavily used in medical imaging; based on CT, MRI



Volumes: Construct. Solid Geometry

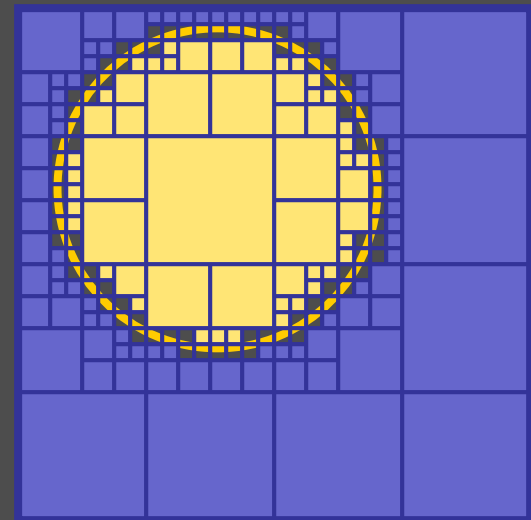
- Boolean operators to combine shapes
- unions, intersections, set differences
- build up CSG trees



courtesy of
Erwin de Groot
and Brian Wyvill

Volumes: BSP and Octrees

- space partitioning: define sub-spaces
- **binary space partitioning**: half-spaces
 - planes define borders between inside and outside; hierarchy
 - only current subspace affected
- **octrees**: partitioning on regular $2 \times 2 \times 2$ grid (= 8)
 - mark cells inside, outside, or subdivide
 - 2D case: quadtrees (2×2)



Transformations

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

Transformations

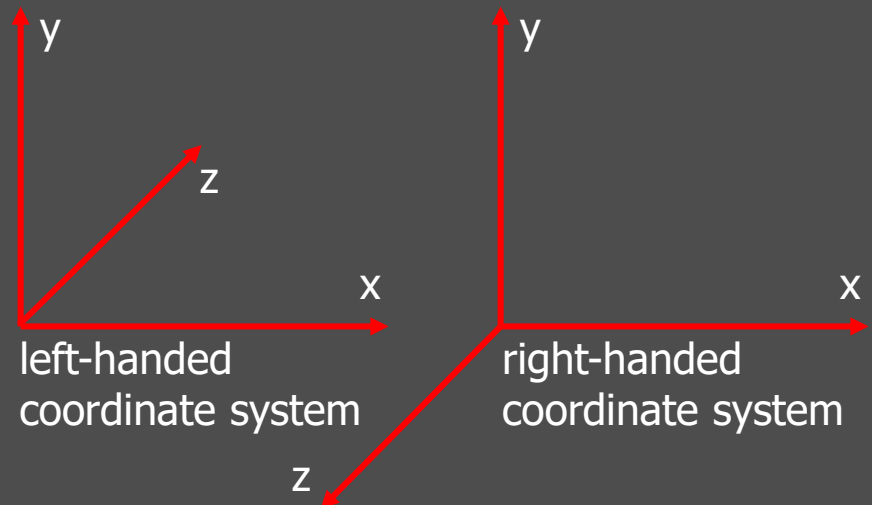
- 3D models represented as points, edges, polygons in 3D coordinate systems
 - object coordinate systems
 - world coordinate system
 - camera coordinate system
 - screen coordinate system
- transformations necessary
- foundation: geometry and linear algebra

Coordinate Systems in CG: 2D & 3D

- two-dimensional



- three-dimensional
 - 2 mirrored systems
 - cannot be matched by rotation
 - we use right-handed

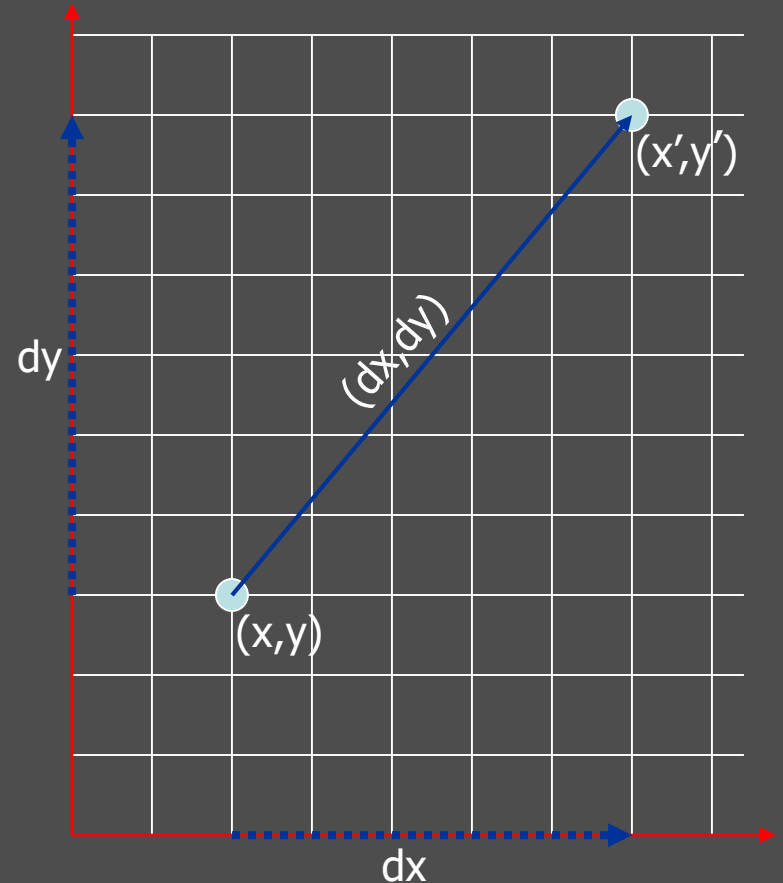


2D Translation

- move point $(x, y)^T$ on a straight line to $(x', y')^T$
- represent translation by a translation vector that is added

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} dx \\ dy \end{pmatrix}$$

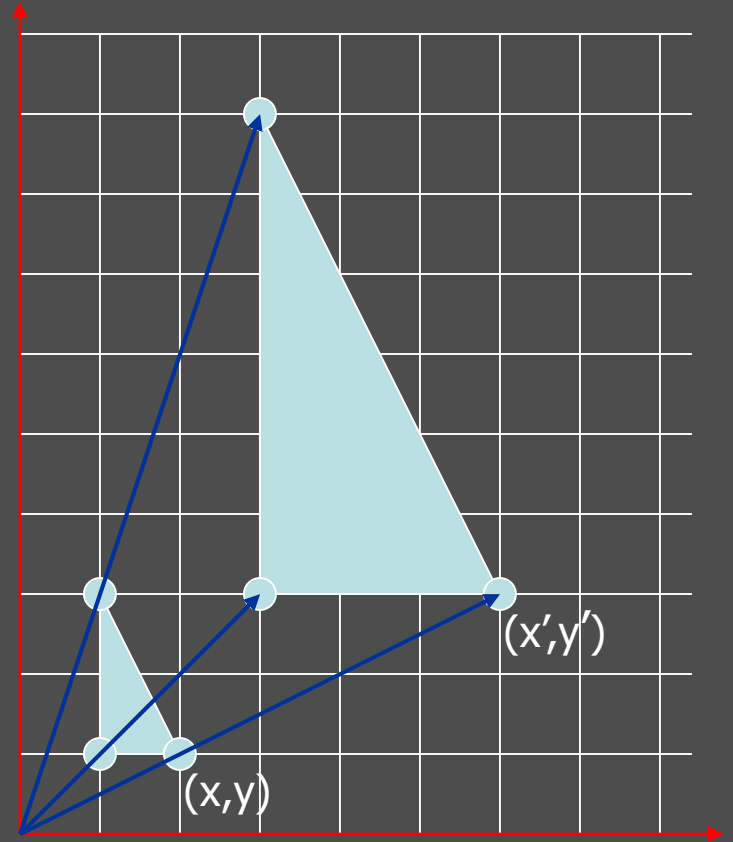
- vector: movement from one point to another



2D Uniform Scaling

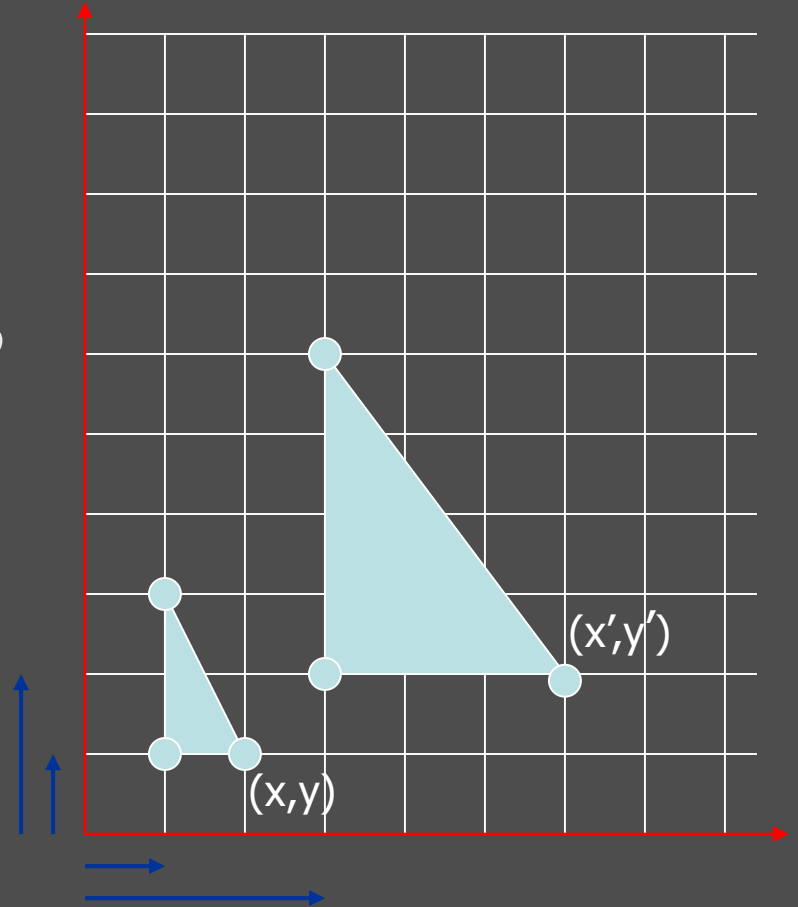
- center of scaling is 0
- scaling uniformly in all directions
- stretching of $(x, y)^T$'s position vector by scalar factor α to get $(x', y')^T$
- mathematically: multiplication with α

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \alpha \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha x \\ \alpha y \end{pmatrix}$$



2D Non-Uniform Scaling

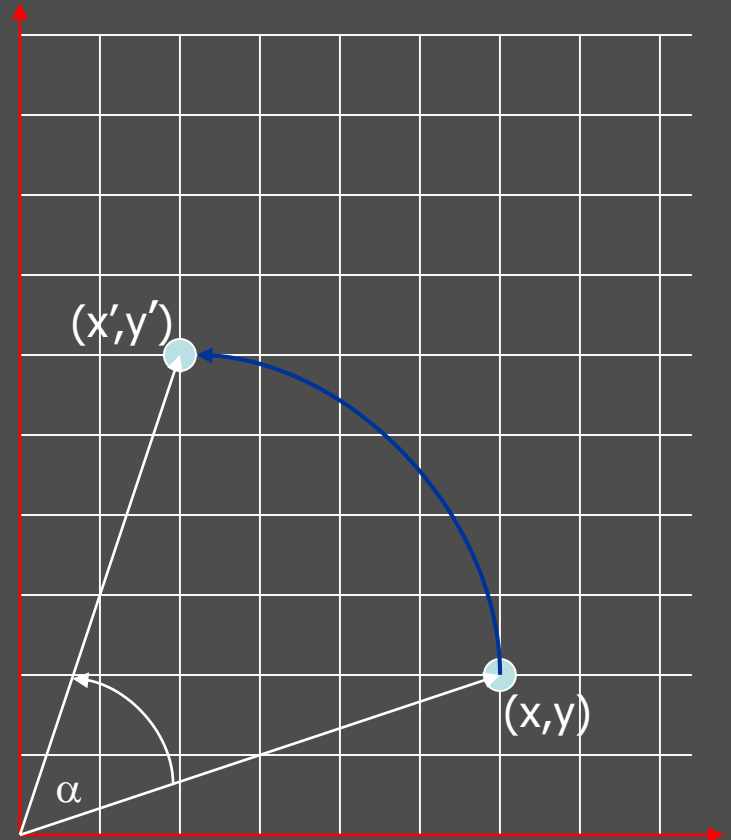
- center of scaling is 0
- scaling in x -direction by α and in y -direction by β (scaling vector $(\alpha, \beta)^T$)
- mathematically: multiplication with α and β according to axis
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \alpha x \\ \beta y \end{pmatrix}$$
- application: mirroring



2D Rotation

- center of rotation is 0
- point $(x, y)^T$ is rotated by an angle α around 0 to obtain $(x', y')^T$
- positive angles α mean counter-clockwise rotation
- $x' = x \cos\alpha - y \sin\alpha$
 $y' = x \sin\alpha + y \cos\alpha$
- matrix multiplication:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Transformations in 2D: Results

- translation: addition of translation vector
- scaling: multiplication of factor(s)
- rotation: matrix multiplication
- problems:
 - non-uniform treatment of transformations
 - no way to combine N transformation into one
- idea: all transformations as matrix multiplications!
- only scaling and translation to do

Scaling using 2D Matrix

- general 2D matrix multiplication format

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- scaling formula

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \alpha x \\ \beta y \end{pmatrix} \text{ (possibly with } \alpha = \beta \text{)}$$

- scaling as matrix multiplication

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Translation using 2D Matrix

- general matrix multiplication format

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- translation formula

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} dx \\ dy \end{pmatrix}$$

- translation as matrix multiplication

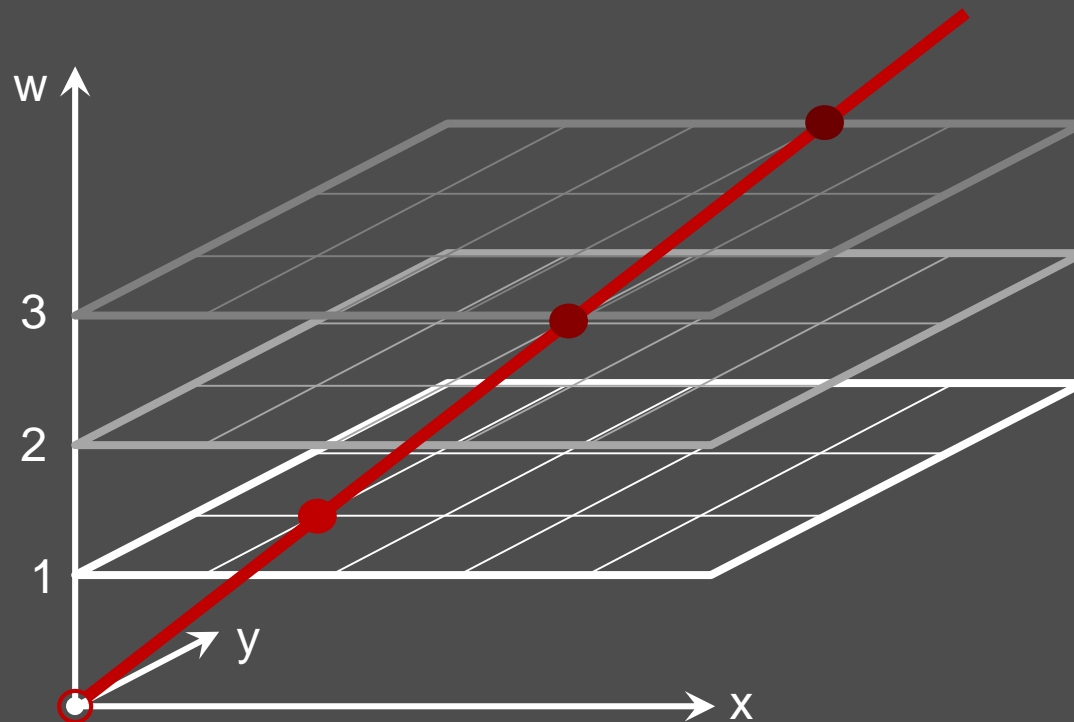
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{not possible in } 2 \times 2 \text{ matrix! } \text{☹}$$

Solution: Homogeneous Coordinates

- add an additional dimension to our vector space A^n : $n \rightarrow n+1$
- $(x, y)^T$ represented as $(wx, wy, w)^T$, $w \neq 0$
- normalized using $w = 1 \rightarrow (x, y, 1)^T$
- each point in A^n is equivalent to a line in homogeneous space A^{n+1} that originates in the origin o but without including o
- homogeneous coordinates are not to be confused with “regular” 3D coordinates!

Homogeneous Coordinates in 2D

- each point in A^n is equivalent to a line in homogeneous space A^{n+1} that originates in the origin o but without including o



Solution: Homogeneous Coordinates

- advantages of homogeneous coordinates
 - uniform treatment of transformations
 - all transformations can be represented
 - combined transformations as one matrix
- procedure: matrix-vector multiplication

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

- goal: derive transformation matrices

Translation in Homogeneous Coords

- general matrix multiplication format

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

- translation formula

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} dx \\ dy \end{pmatrix}$$

- translation as matrix multiplication

$$\begin{pmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Scaling in Homogeneous Coords

- scaling as matrix multiplication

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- scaling as homogeneous matrix multiplication

$$\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Rotation in Homogeneous Coords

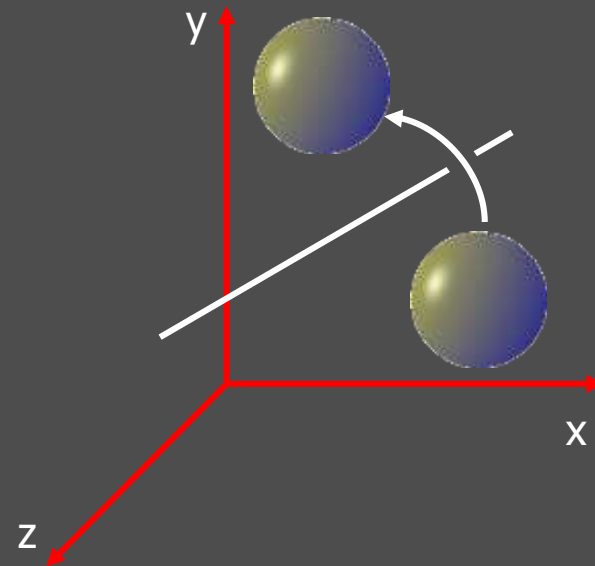
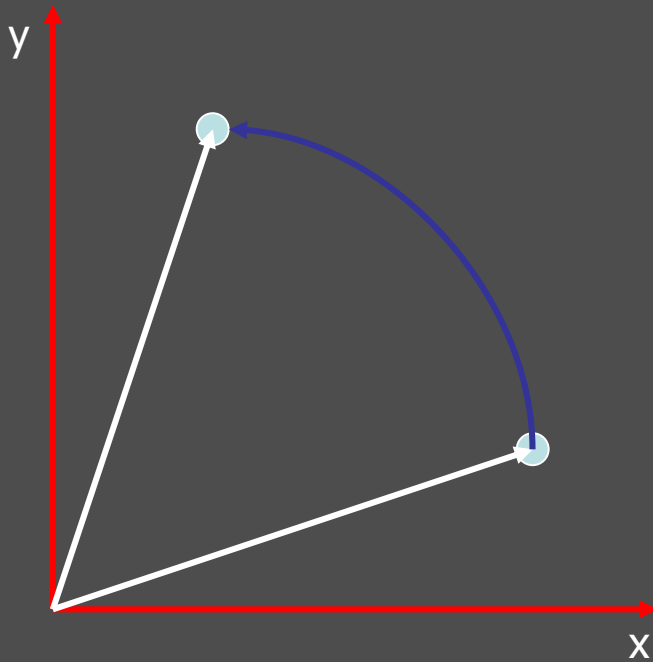
- rotation as matrix multiplication

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- rotation as homogeneous matrix multiplication

$$\begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Transformations in 3D Space



Geometric Transformations in 3D

- same approach as in 2D
- also use homogeneous coordinates (for the same reasons)
- vectors/points from 3D to 4D

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- transformation matrices are now 4×4 (instead of 3×3)

Transformation Matrices in 3D

- translation

- translation vector
 $(dx, dy, dz)^T$

$$T = \begin{pmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- scaling

- for uniform scaling

$$s_x = s_y = s_z$$

- otherwise individual factors may differ

- mirroring using factors of -1 and 1 depending on the mirror plane

$$S = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation Matrices in 3D

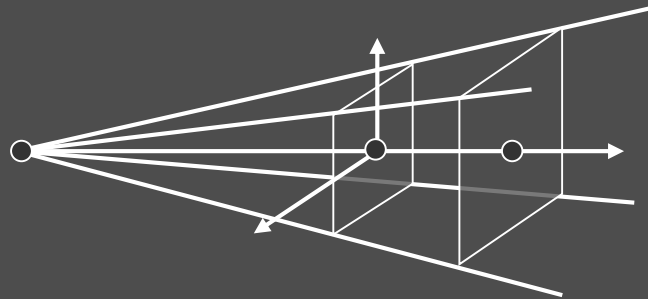
- rotation
 - rotation around 3 axes possible now
 - each has individual rotation matrix
 - rotation around positive angles in right-handed coordinate system
 - rotation axis stays unit vector in matrix

$$R_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_y = \begin{pmatrix} \cos \alpha & 0 & \sin \alpha & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \alpha & 0 & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 & 0 \\ \sin \alpha & \cos \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Viewing and Projections



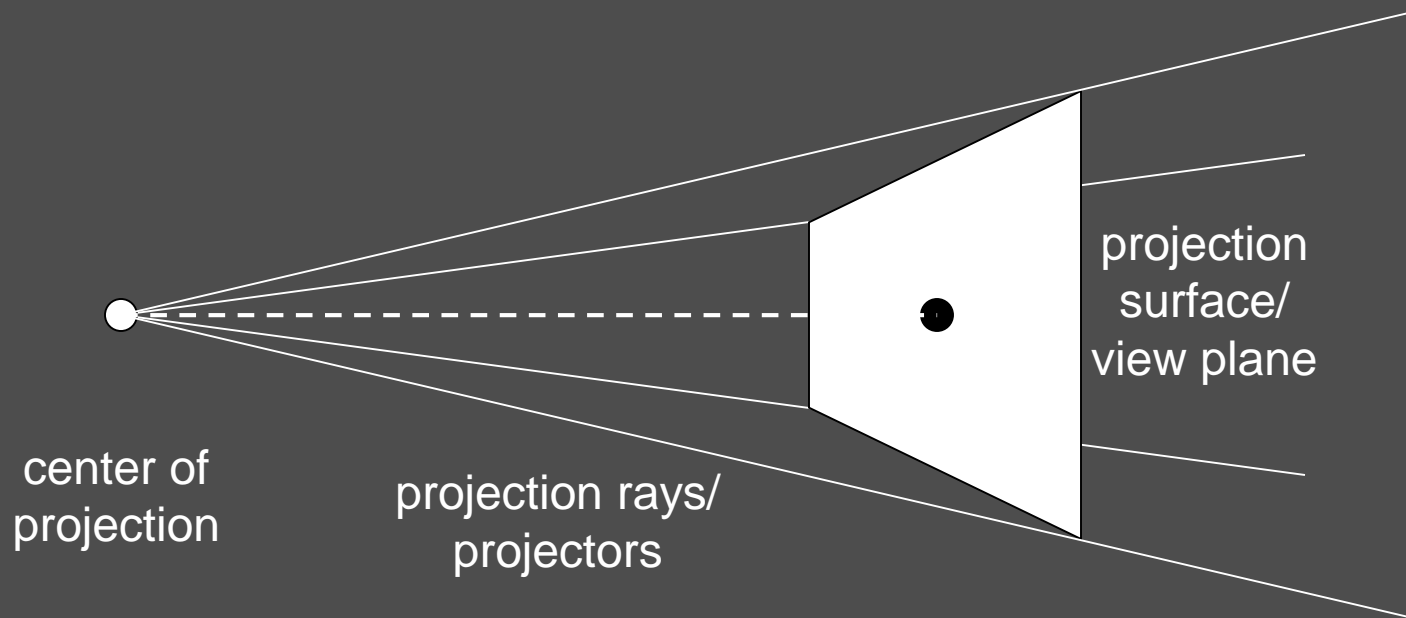
Model-View Transformation

- model-view transformation steps:
 1. translate object origin to camera location
 2. rotate to align coordinate axes
 3. possibly also scaling
- this process is used in OpenGL:
no explicit world coordinates!
- object & camera locations and orientations may be specified in a world coordinate system (e.g., in modeling systems)
- allows object hierarchies with animation
- separate from projection (next)

Projections

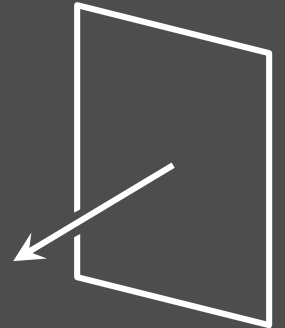
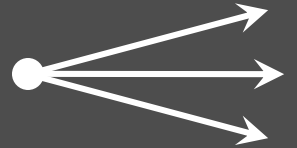
planar projection:

- projection rays are straight lines
- projection surface/view plane is planar
- projections of straight lines are also straight

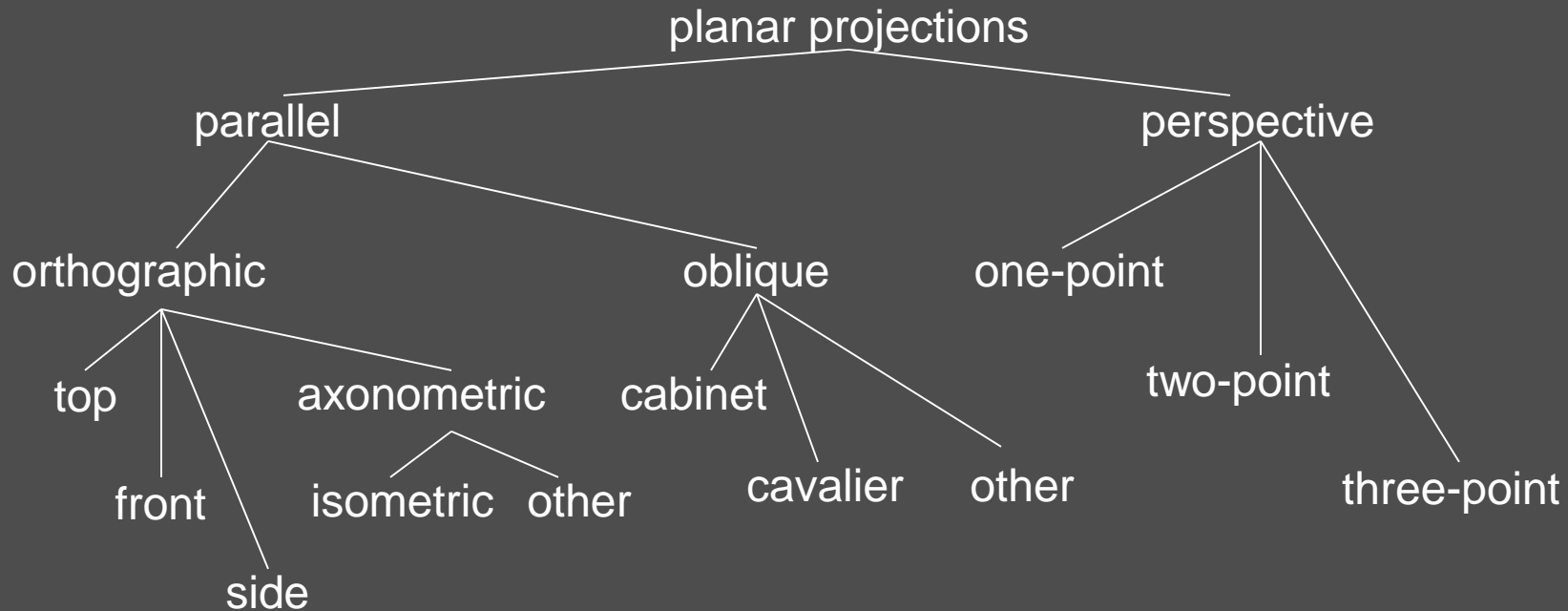


Introduction – Terms

- parallel projection: characterized by **direction of projection** (dop)
- perspective projection: **center of projection** (cop)
- projection on **view plane**
- vector perpendicular to view plane: **view plane normal** (vpn)
- rays characterizing projection: **projectors** (parallel or diverging from cop)



Classification of Planar Projections



- parallel projections:
all projectors parallel to each other
- perspective projections:
projectors diverge from cop

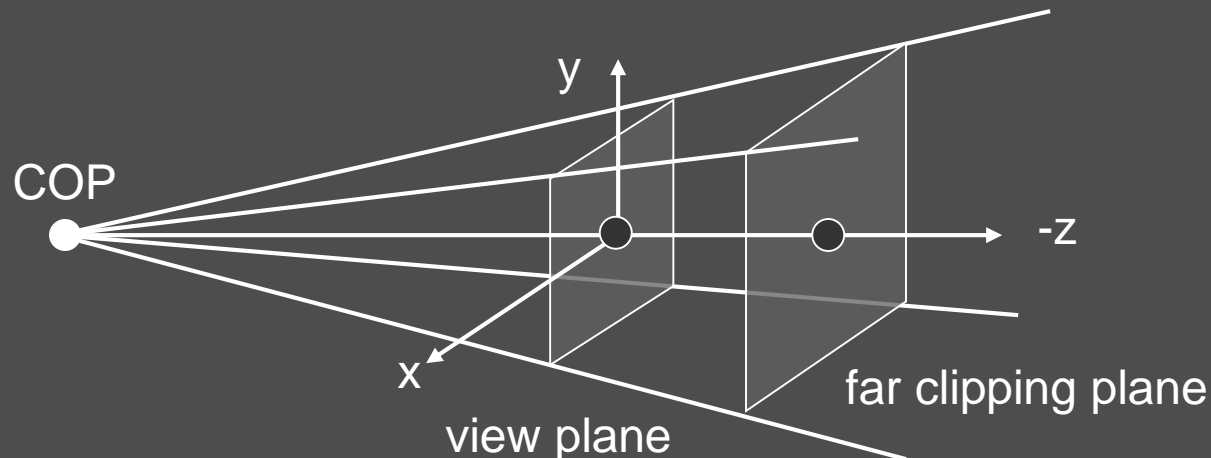
Perspective Projections: Camera Model

- inspired by real cameras
- parameters:
 - position, orientation
 - aperture
 - shutter time
 - focal length, type of lens (zoom vs. wide)
 - depth of field
 - size of resulting image
 - aspect ratio (4:3, 16:9, 1.85:1, 2.35:1, 2.39:1)
 - resolution (digital cameras)



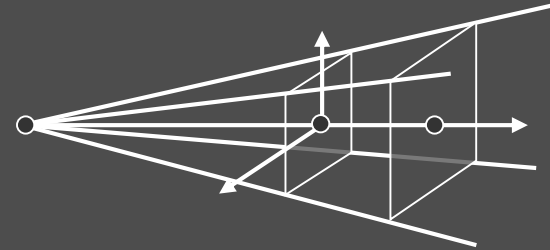
Perspective Projections: Camera Model

- simplified model in CG
 - position: point in 3D (= cop)
 - view direction: vector in 3D (vpn)
 - image specification: viewport
 - clipping planes for cutting off near and far objects (near and far clipping plane)

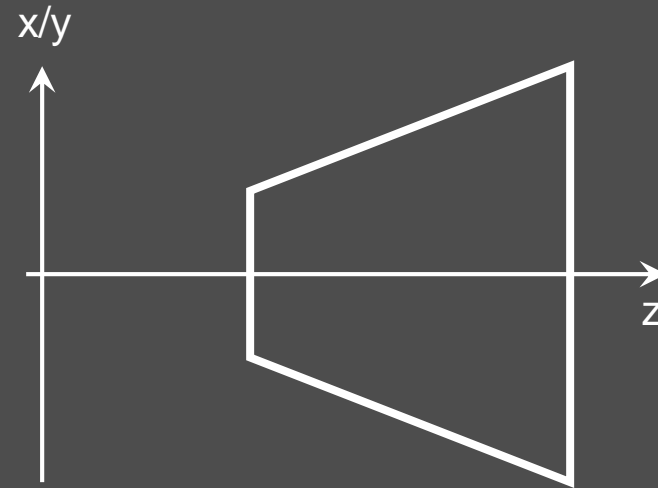
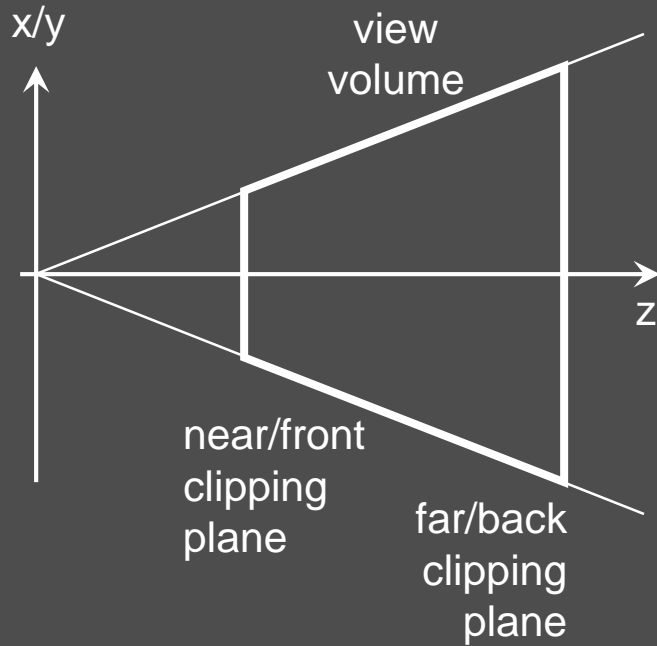


Perspective Projections: Camera Model

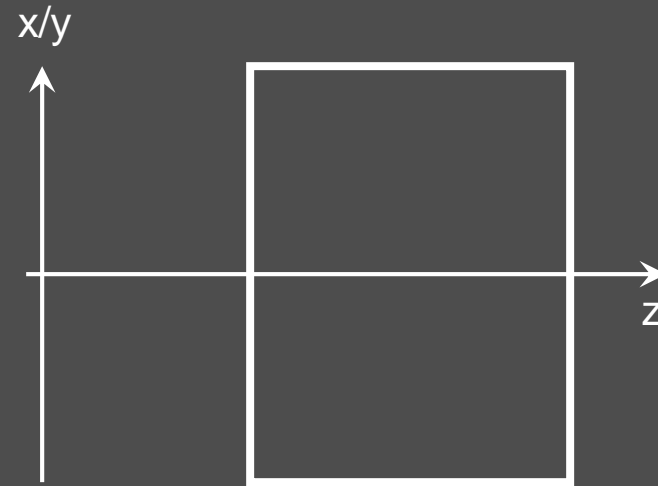
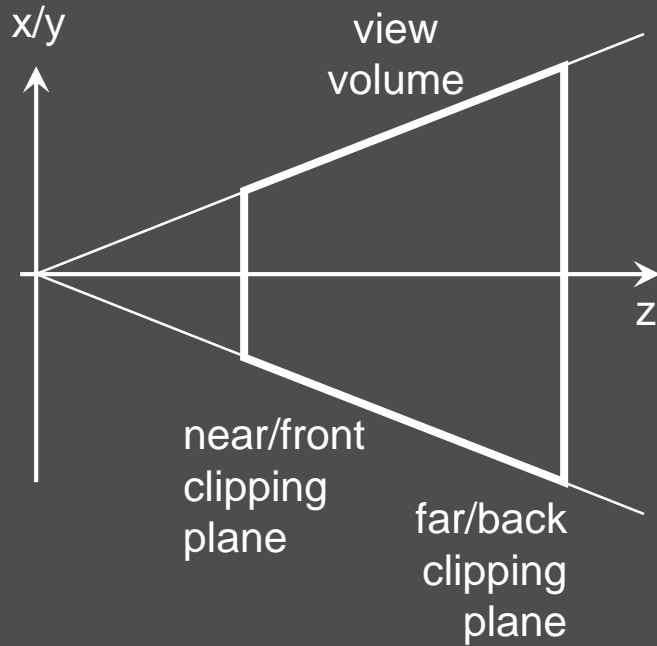
- differences between real and CG camera?
 - position of view plane w.r.t. COP
 - orientation of the image
 - type of camera (pinhole vs. lens)
 - type of “refraction”
 - lens effects (lens flare)
 - depth of field: none vs. existing
 - types of possible projections
 - picture taking times: 0 vs. >0
 - existence of far clipping plane
 - shape of view volume



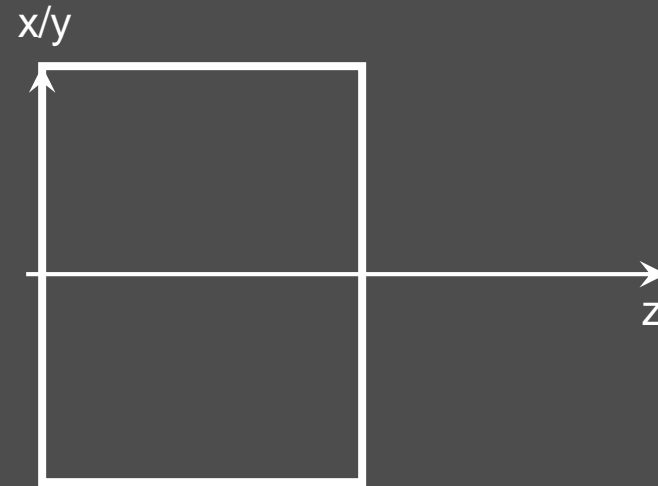
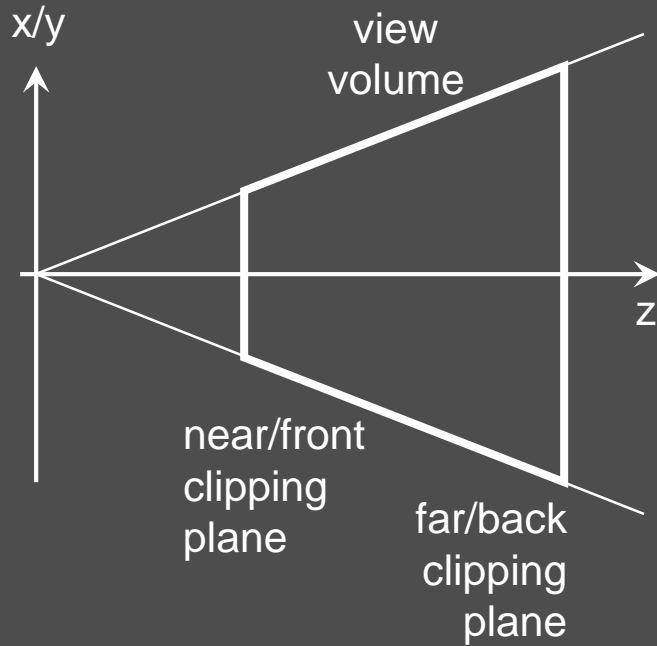
Projections: Canonical View Volumes



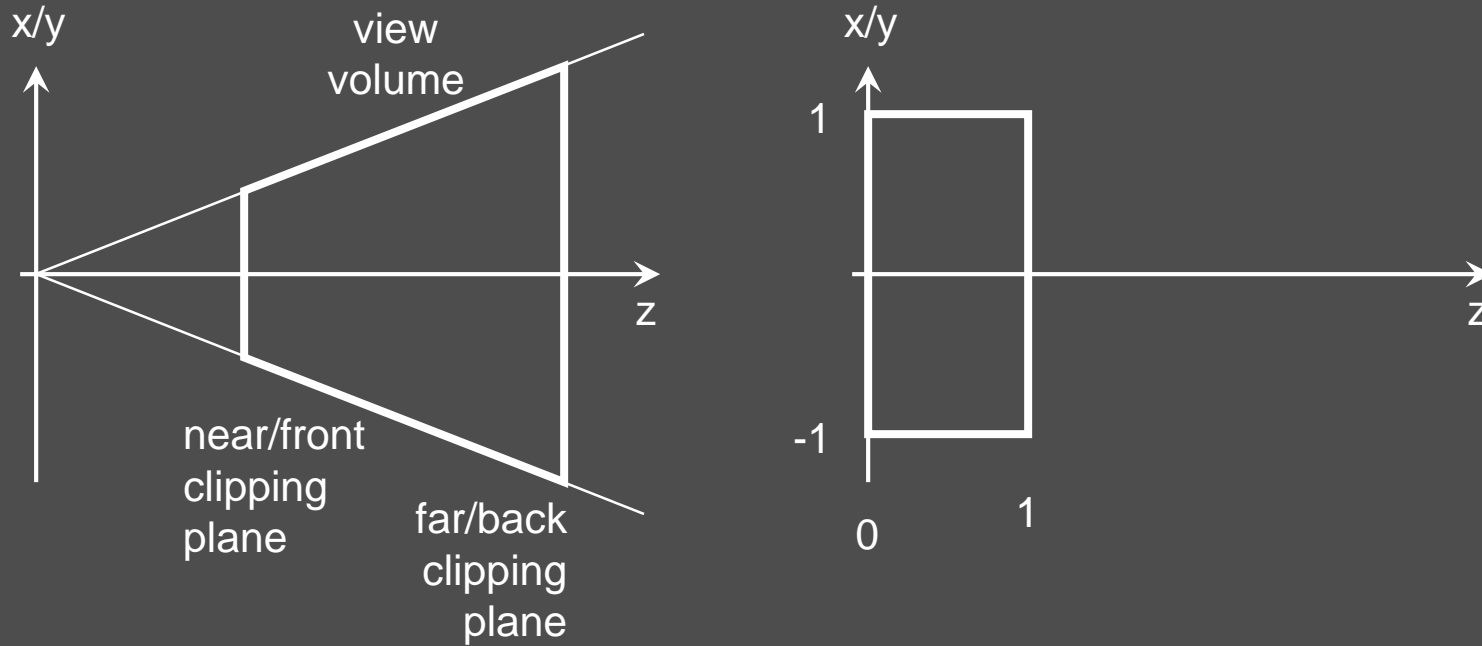
Projections: Canonical View Volumes



Projections: Canonical View Volumes

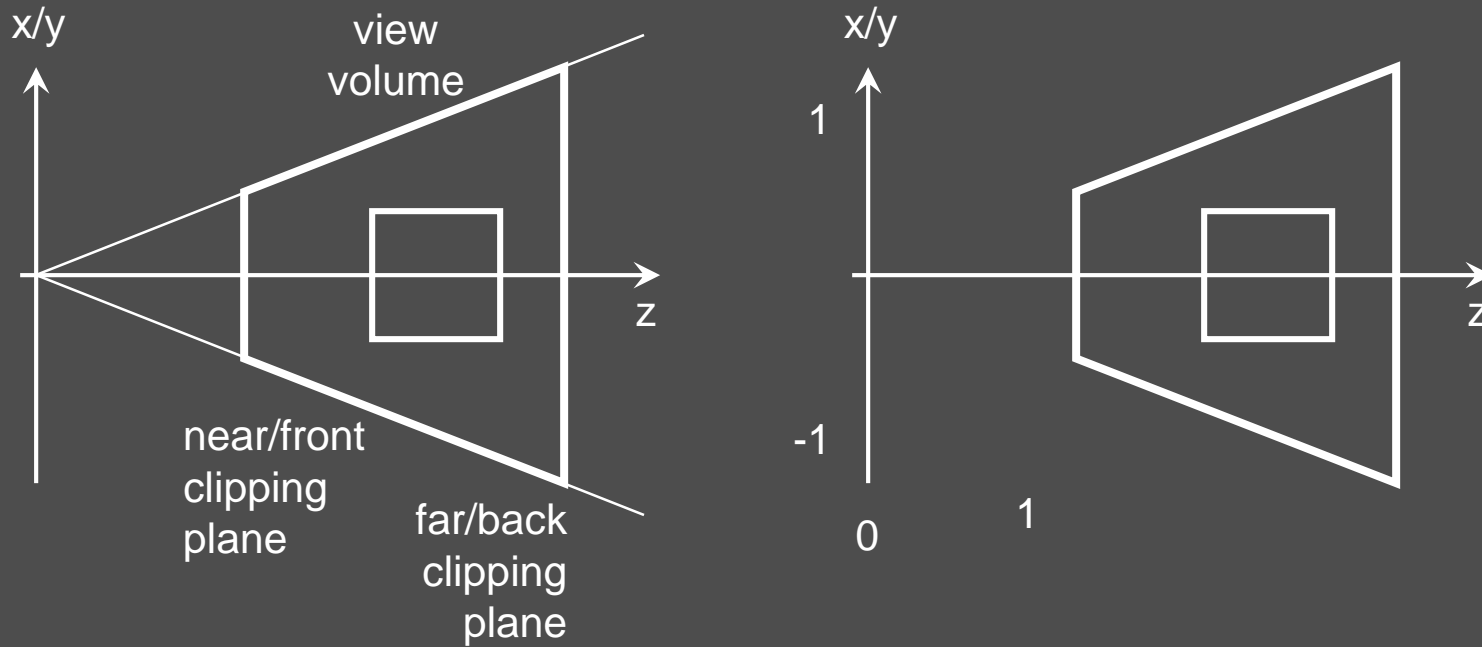


Projections: Canonical View Volumes



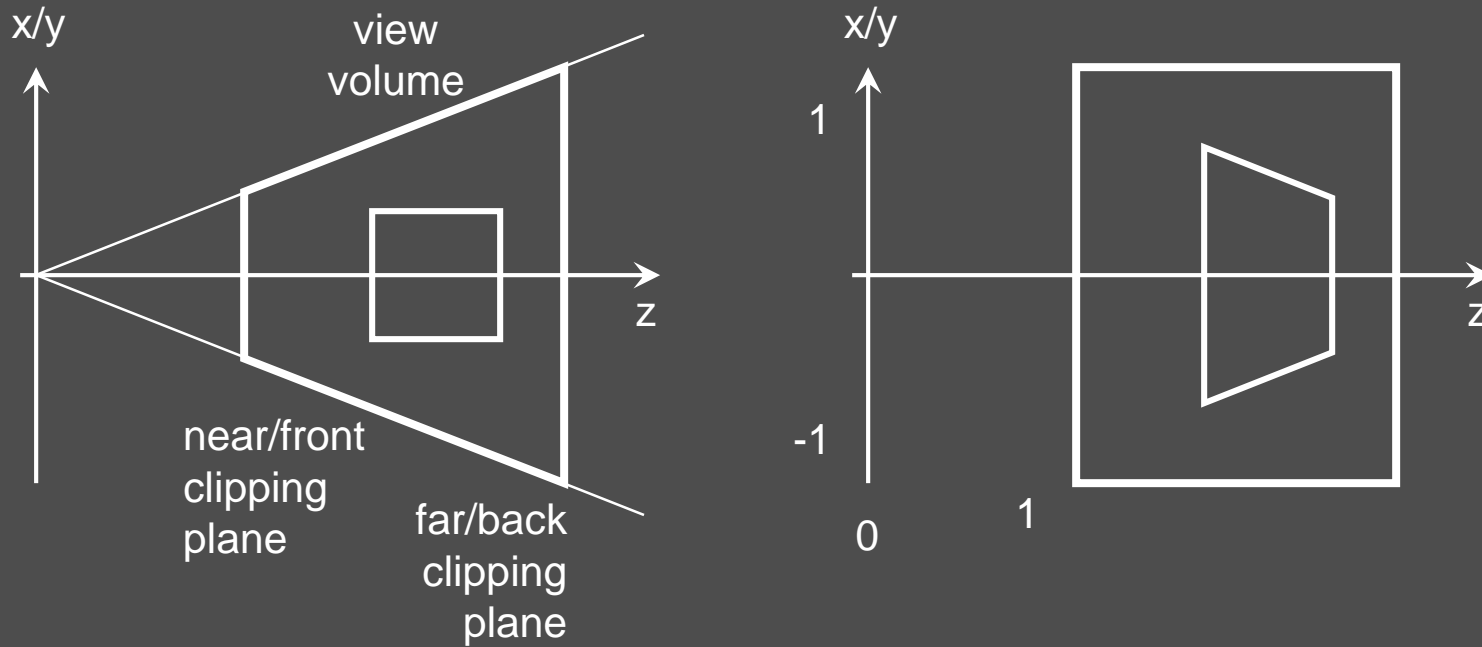
- shearing, translation, scaling
- view volume is only implicitly transformed: included objects are actually transformed!

Projections: Canonical View Volumes



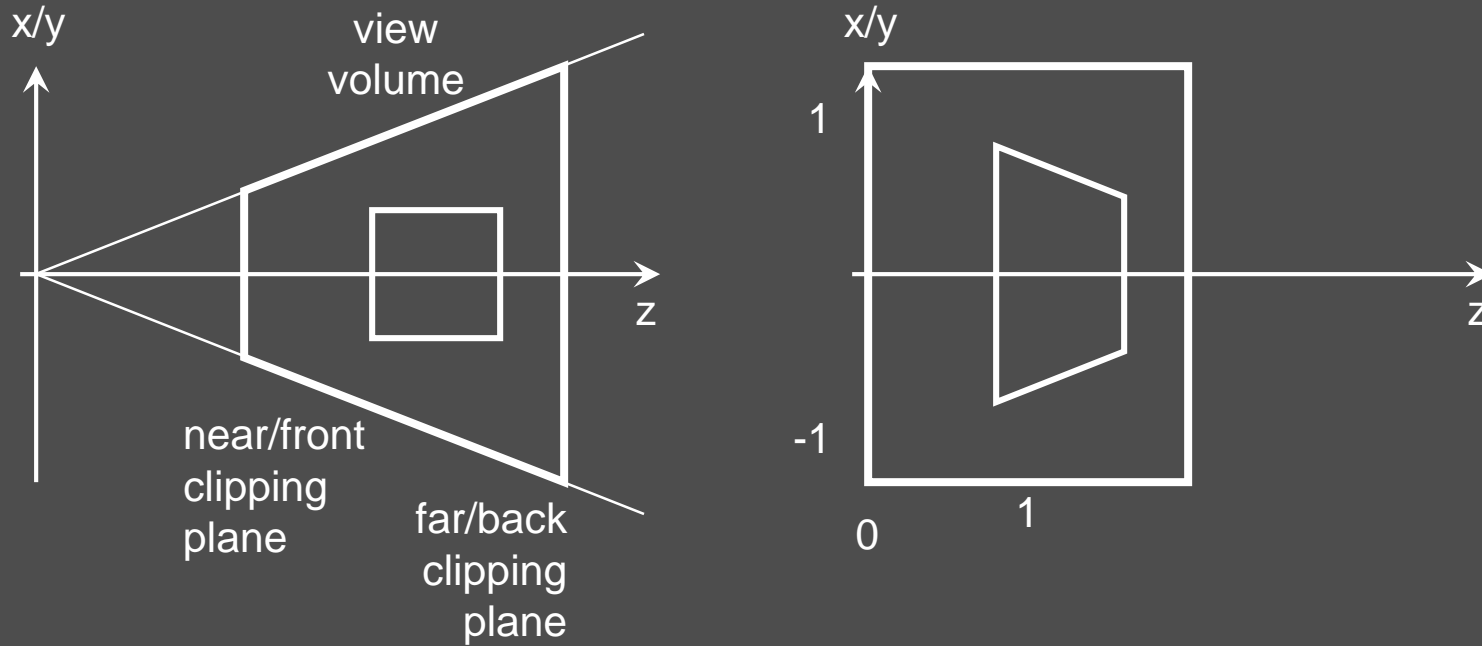
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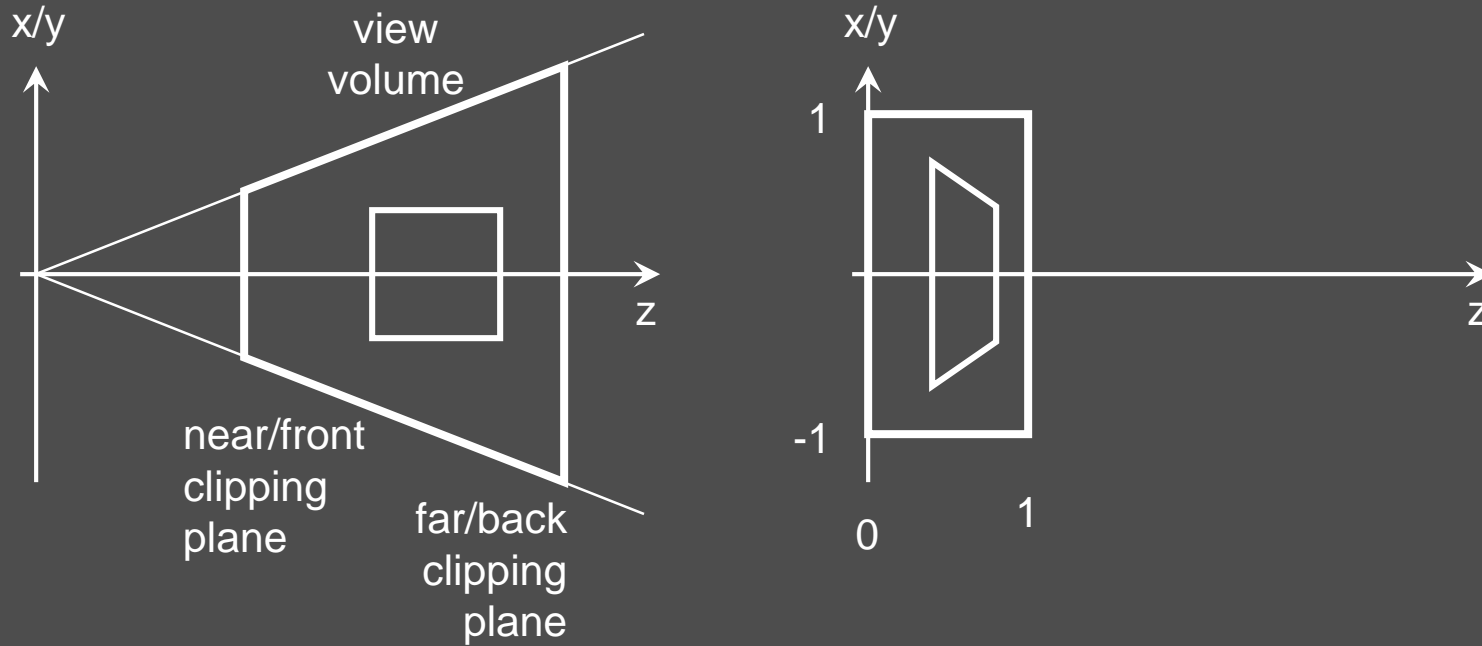
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Projections: Canonical View Volumes



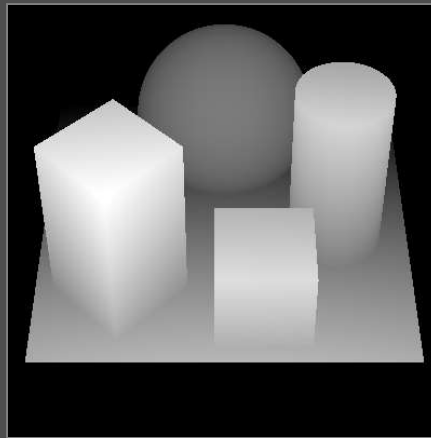
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Projections: Canonical View Volumes



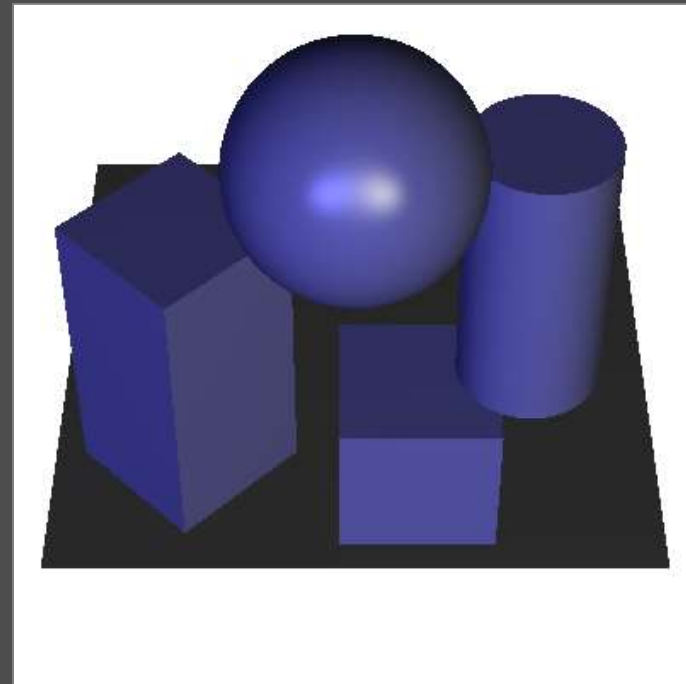
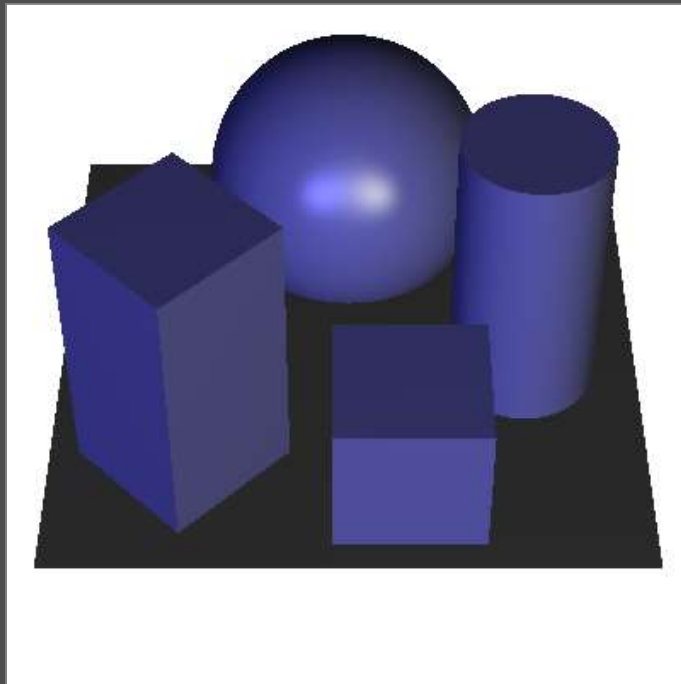
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Hidden Surface Removal



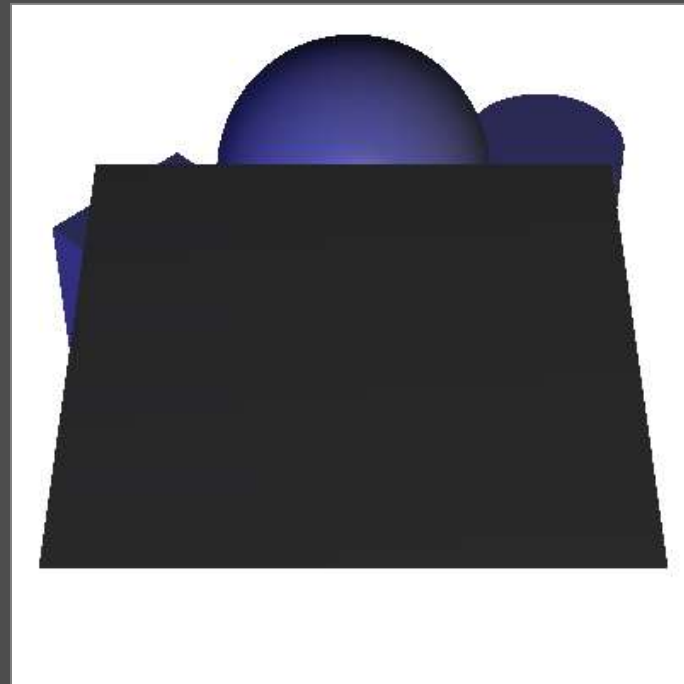
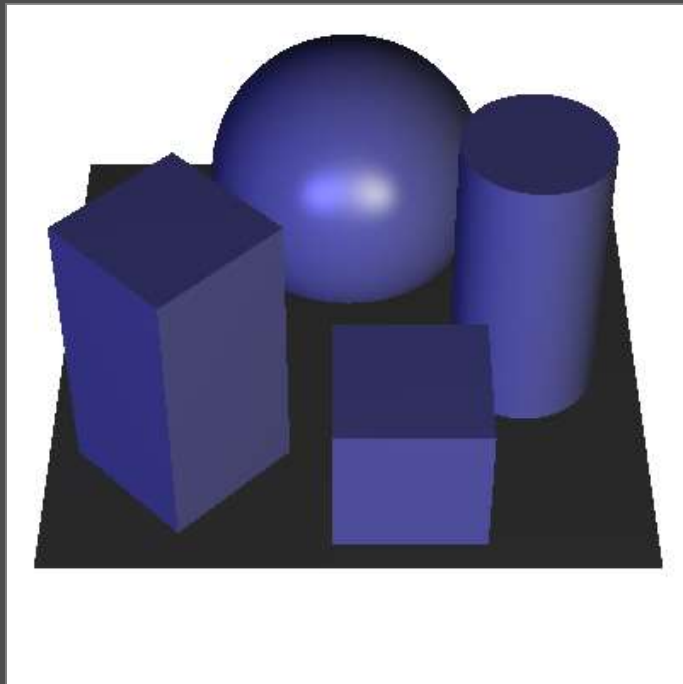
Hidden Surface Removal: Motivation

- goals:
 - model parts independently processed
 - at the same time: show front parts only
 - avoid unnecessary processing



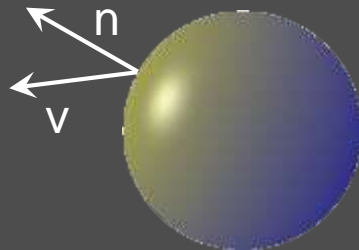
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Back Face Culling

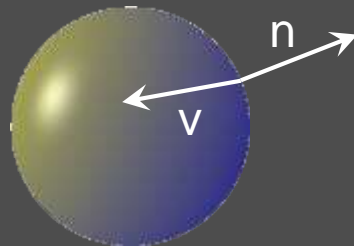
- back faces (usually) not visible
- remove these: reduction of computation
- removal early in the pipeline
- reduction of polygon count by approx. $\frac{1}{2}$ of the total polygon number
- computation: compare dot product of surface normal with view direction with 0



$$n \cdot v > 0$$

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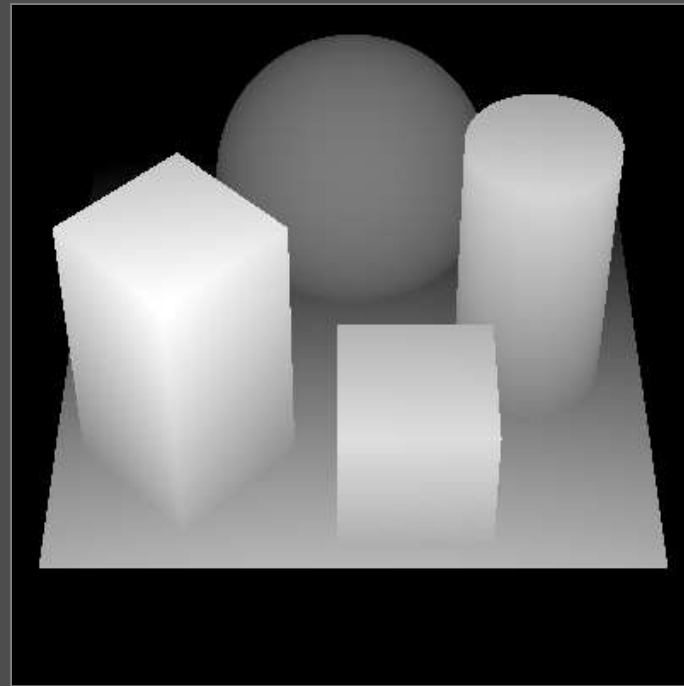
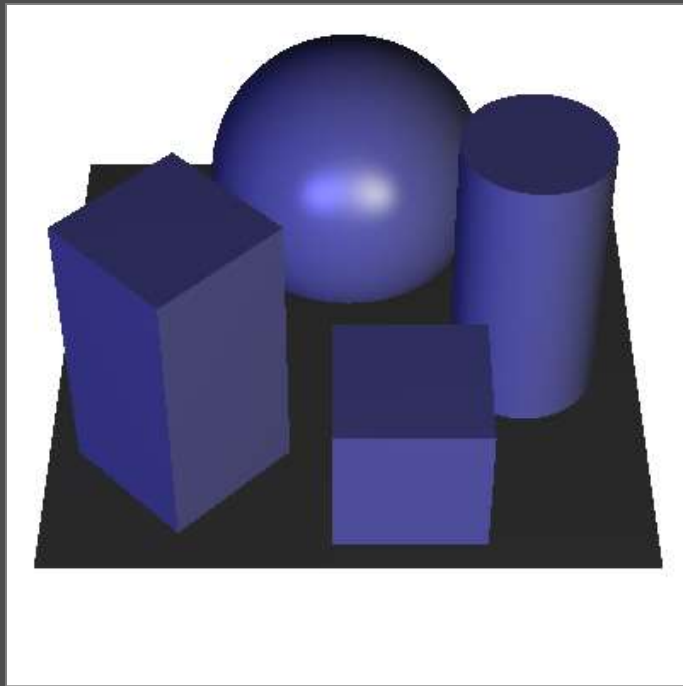
$$n \cdot v < 0$$

z-Buffering

- idea:
 - rendering from back to front (Painter's algorithm)
 - avoid need to sort & problems with cyclic triangles
- realization by
 - trading speed for memory usage
(memory is cheap [now anyway], time is not)
 - trading speed for accuracy
(only compute what we really need/want)

z-Buffering

- introduce new pixel buffer: z-buffer (in addition to the frame buffer for image)
- z-buffer stores z-values of pixels



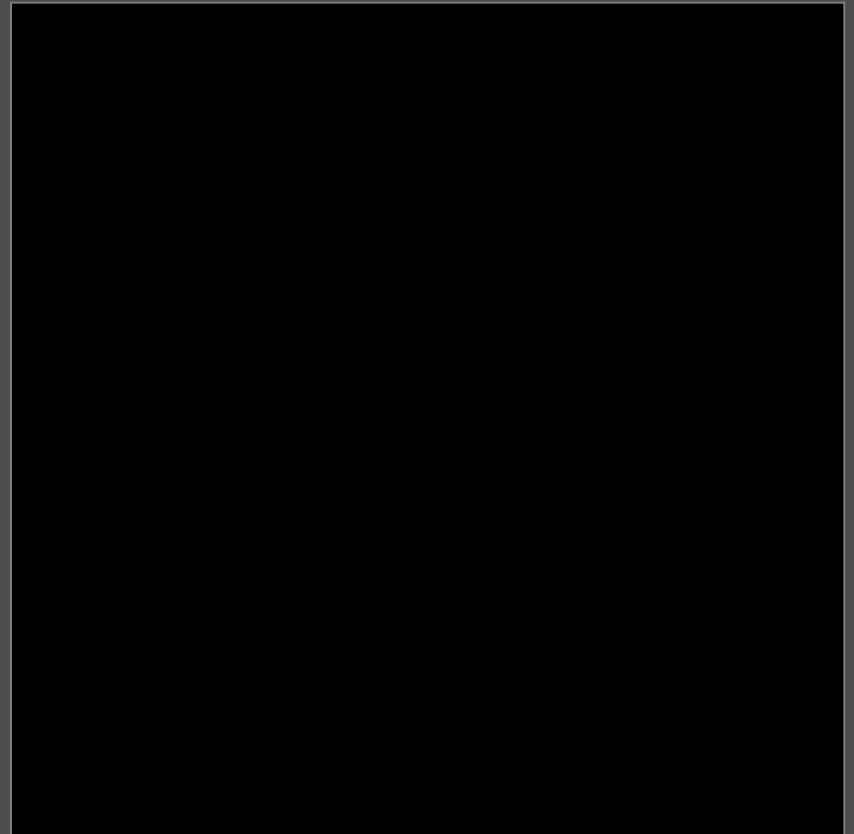
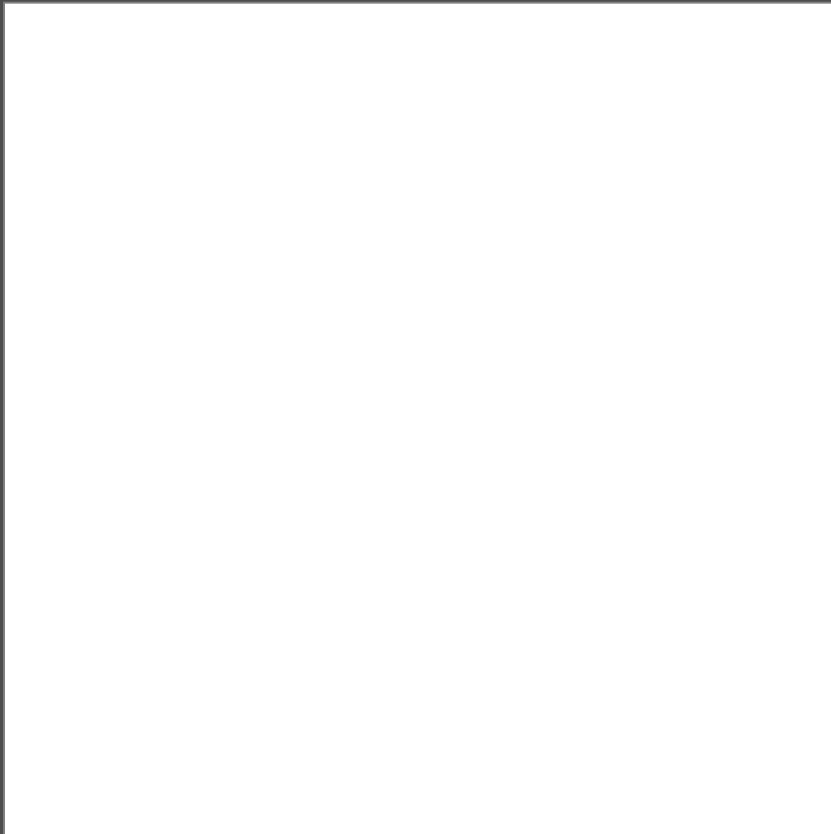
z-Buffering

- treat each primitive (triangle) individually:
scene → objects → triangles → pixels
- use *z*-buffer data to determine if a new triangle is (partially) hidden or not
- at each time, the part of scene that has been processed thus far is correctly displayed

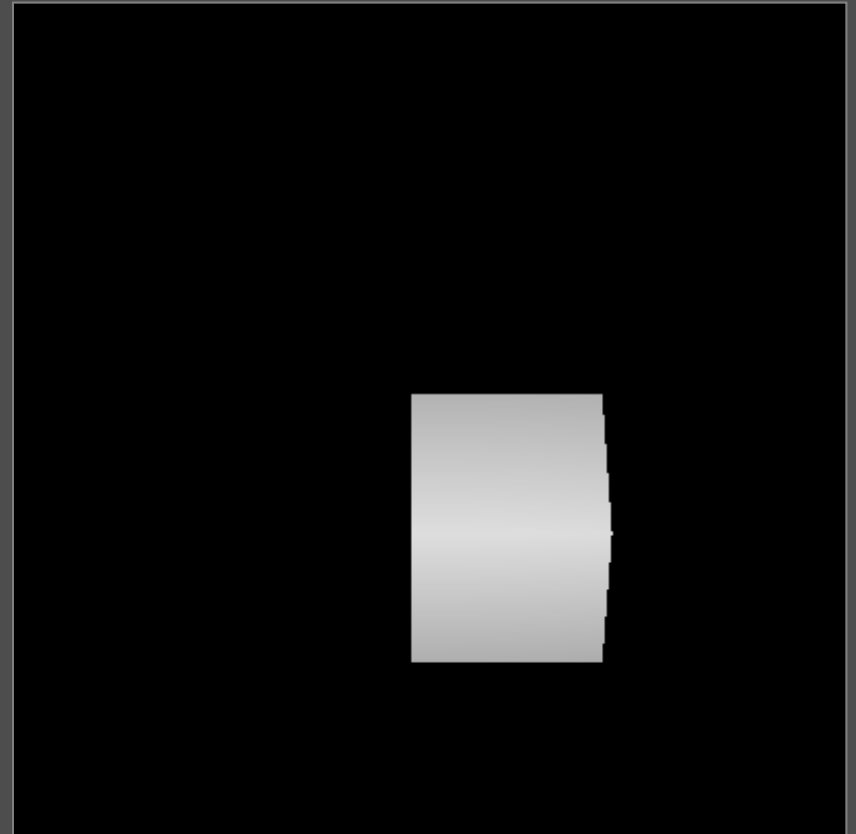
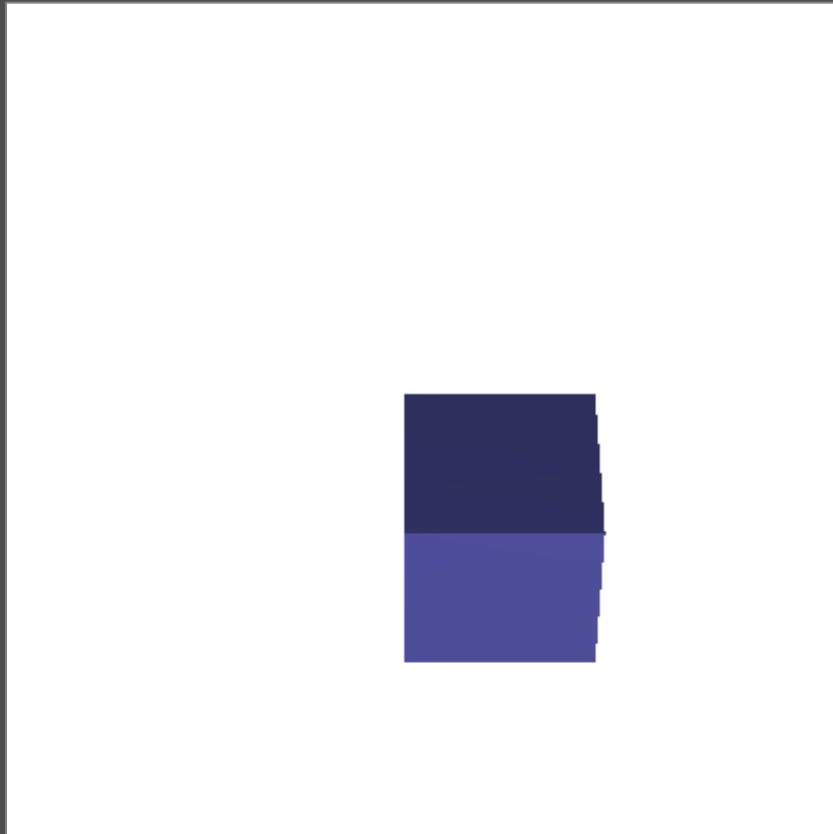
z-Buffering: Algorithm

- initialize z -buffer and frame buffer
- for each triangle
 - project all vertices of the triangle
 - interpolate z -values for each pixel (scan line)
 - before shading a pixel, test if its z -value is closer to camera (i.e. higher) than the current z -buffer value
 - if so: update z -buffer value and shade pixel
 - otherwise: discard pixel and continue
- after scene processing z -buffer contains depth map of scene

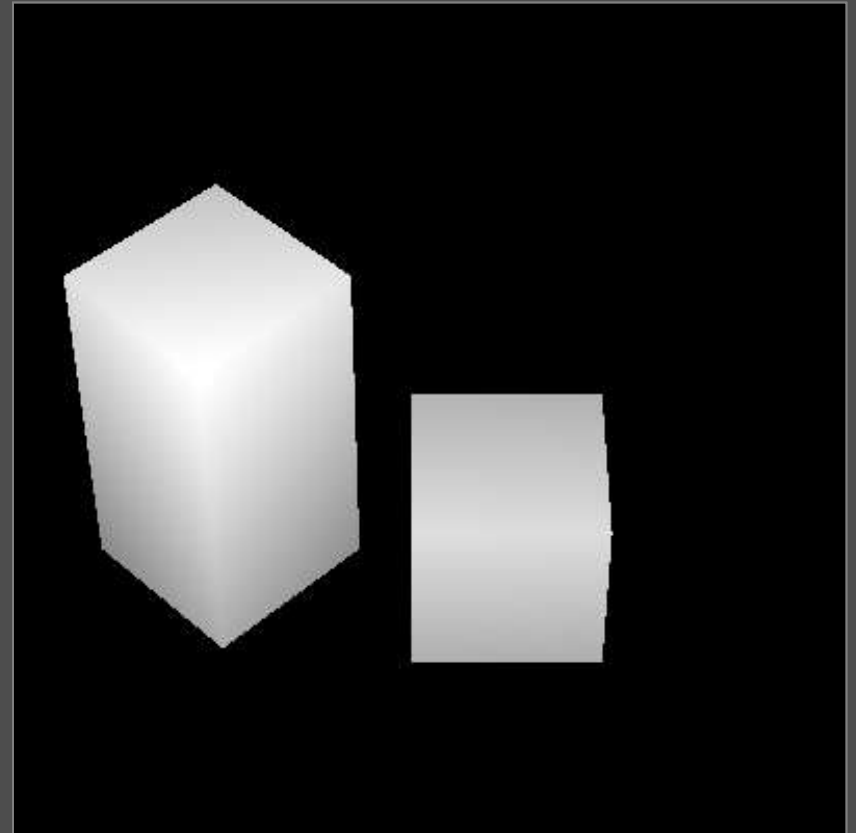
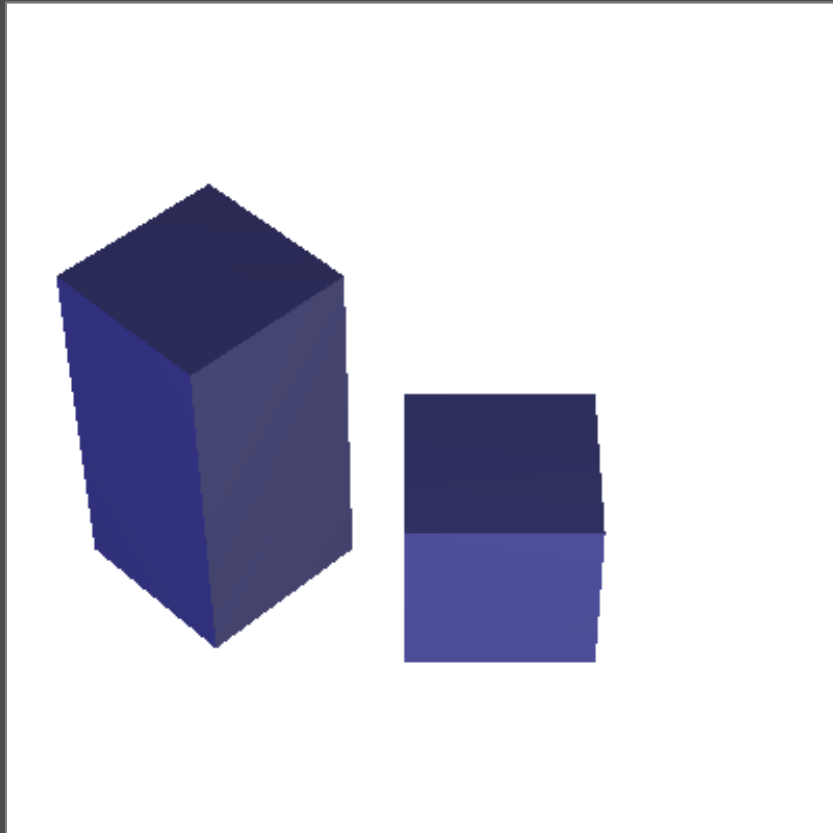
z-Buffering: Example (by object)



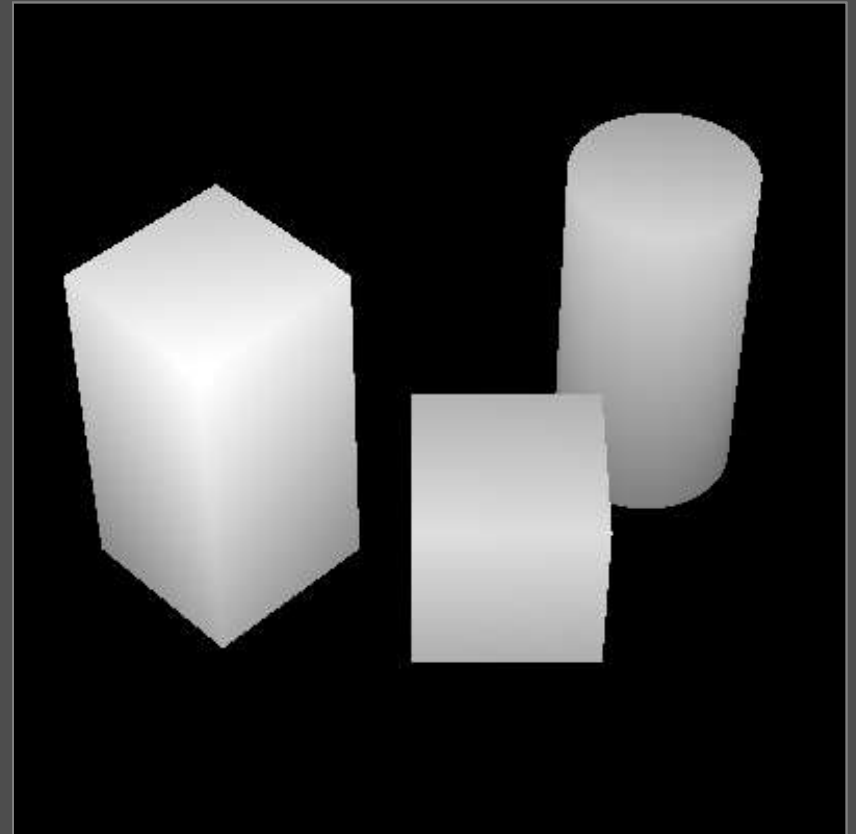
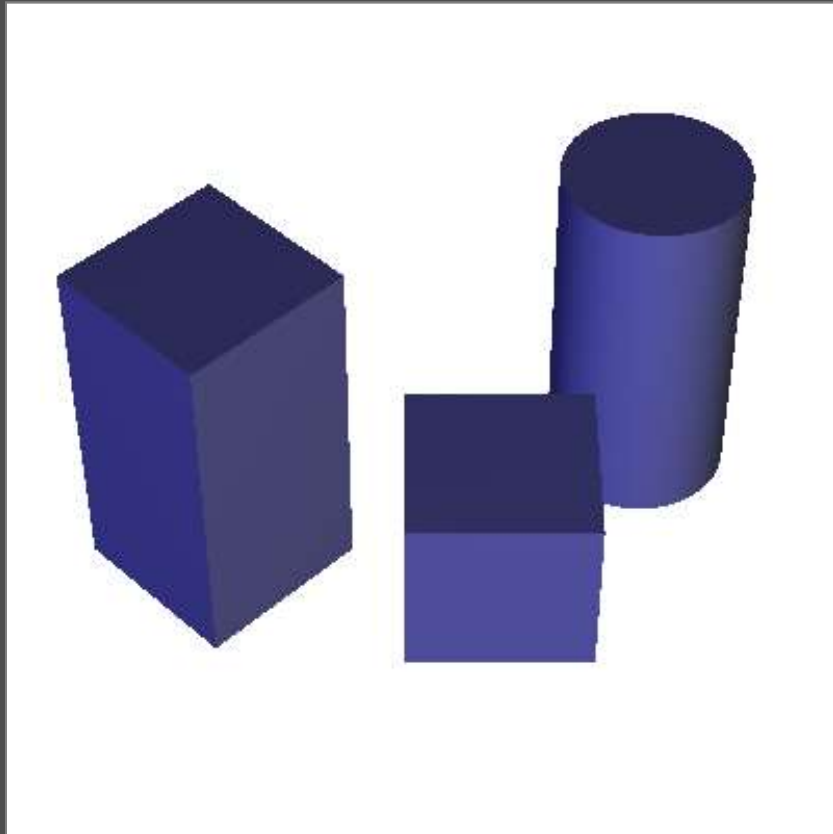
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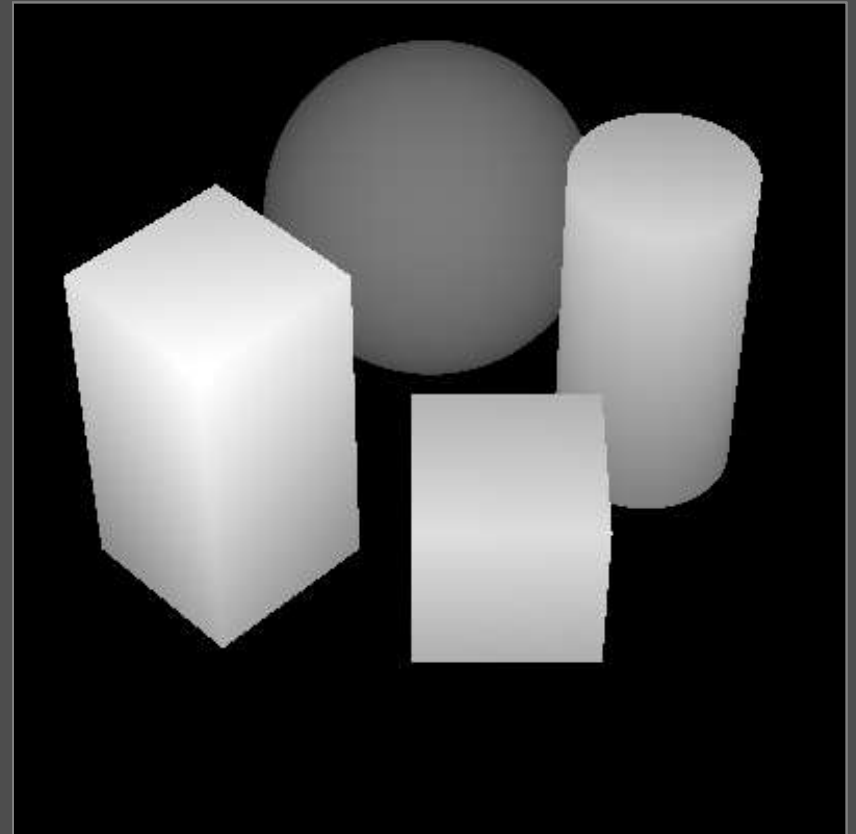
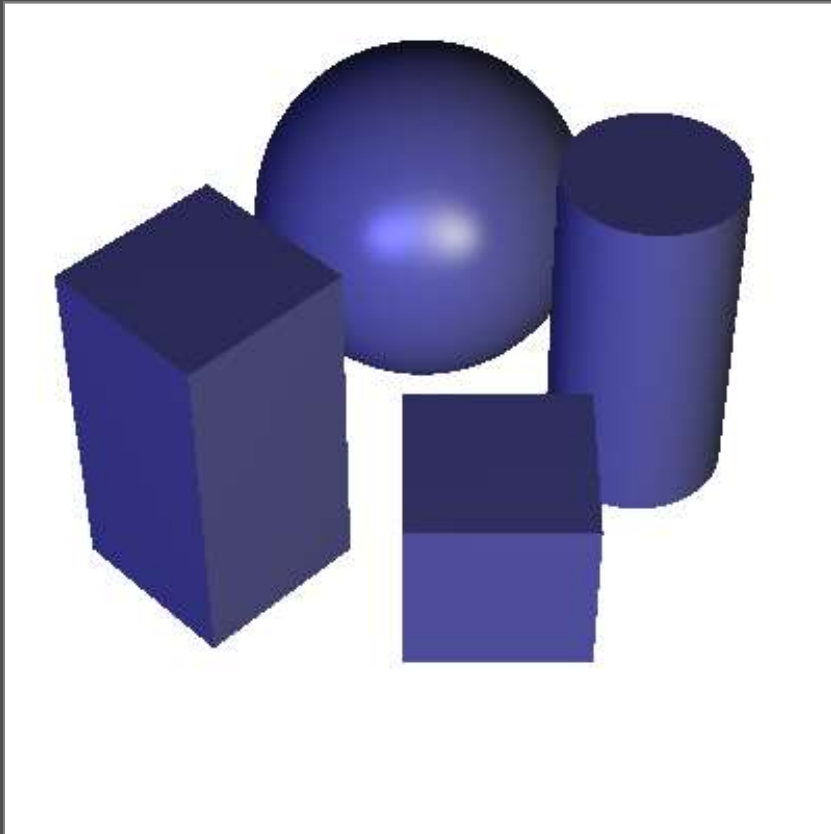
z-Buffering: Example (by object)



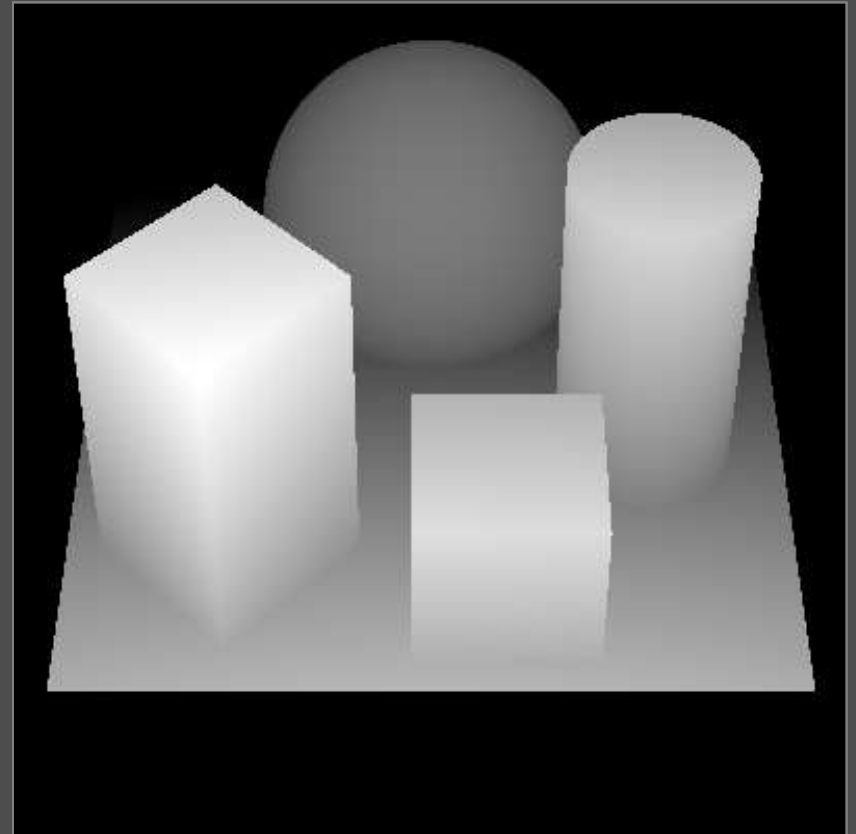
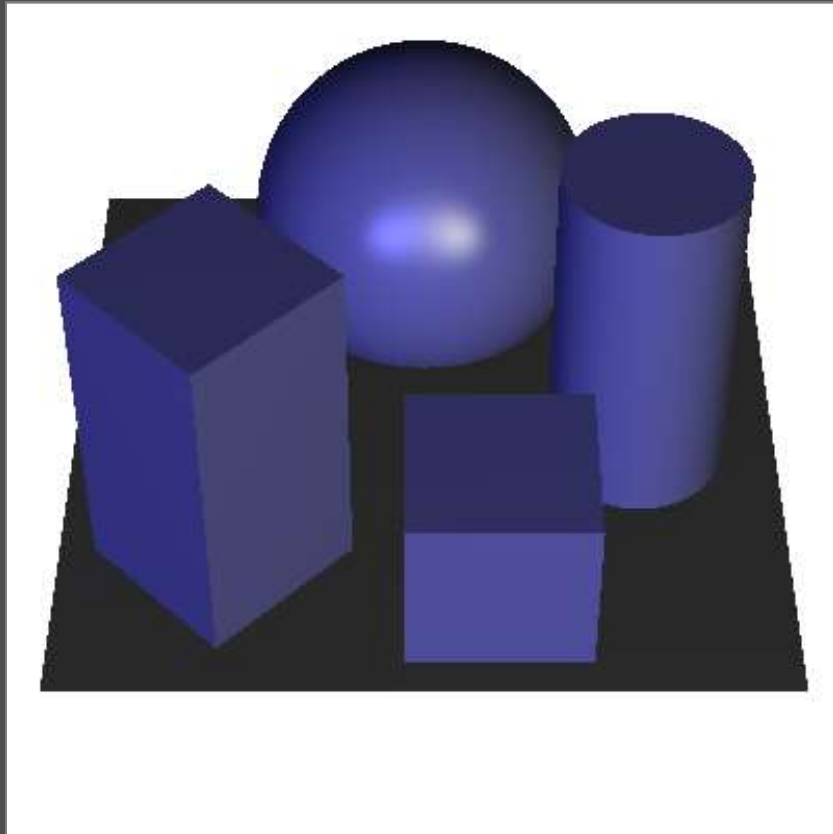
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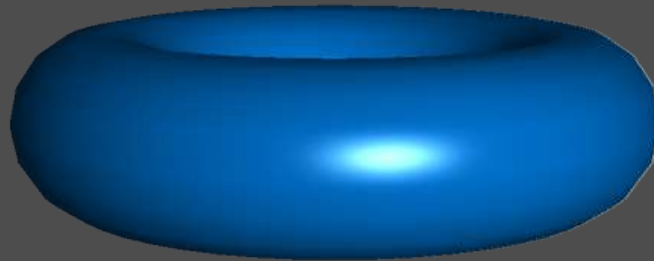
z-Buffering: Example (by object)



z-Buffering

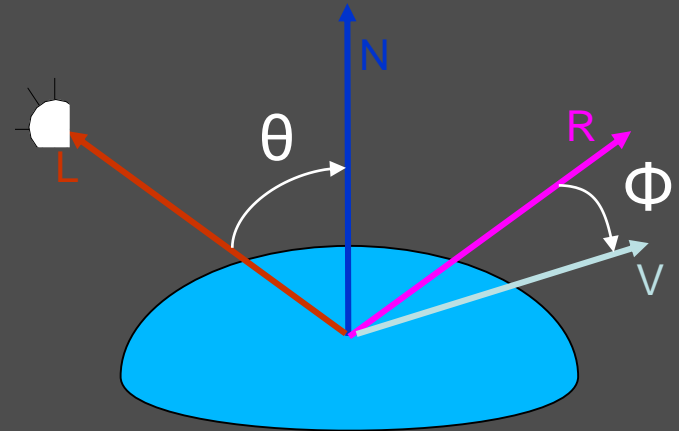
- advantages
 - can be implemented in hardware
 - can process infinitely many primitives
 - does not need sorting of primitives
(only need to know distance to camera)
 - can handle cyclic and penetrating triangles
- disadvantages
 - needs memory to keep all z -values
(image size @ 8bit or 16bit)
 - cannot handle transparency properly

Illumination and Shading



Light Interaction with Surfaces

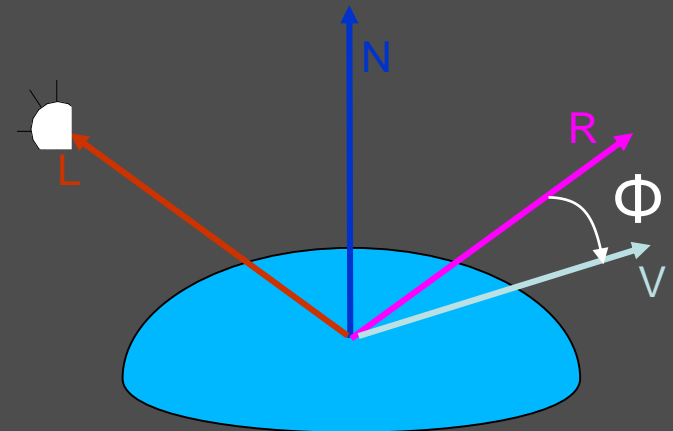
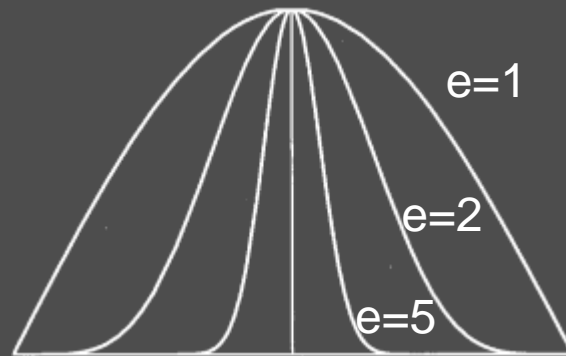
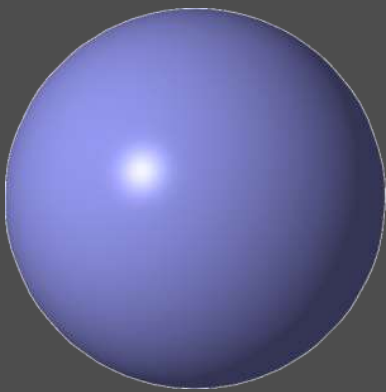
- angle θ between L & N determines diffuse reflection
- reflection angle equals θ
- angle Φ between R & V determines perceived brightness
- maximal reflection if $R = V$ ($\Phi = 0$)



L – vector to light source
N – surface normal vector
R – reflected light ray
V – vector to viewer/observer

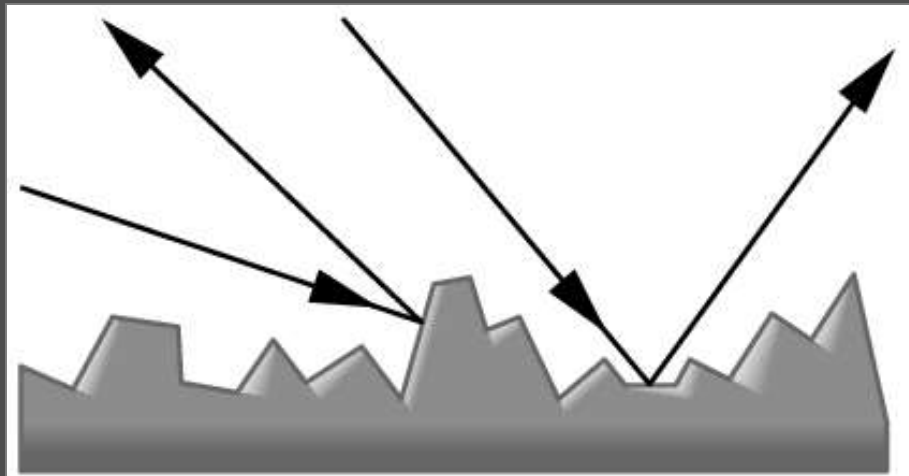
Light Interaction with Surfaces

- **directed reflection:** reflection only for small Φ : smooth surfaces
- light attenuation depending on angle Φ : shininess
- modeled using cosine function and exponent: $I \sim \cos(\Phi)^e$ (e.g., metals: $e \approx 100$)
- physical reality: anisotropic materials



Light Interaction with Surfaces

- **diffuse reflection:** equal reflection in all directions on rough surfaces, depends only on θ and not observer – examples?
- due to light scattering on rough surfaces on randomly oriented microscopic facets



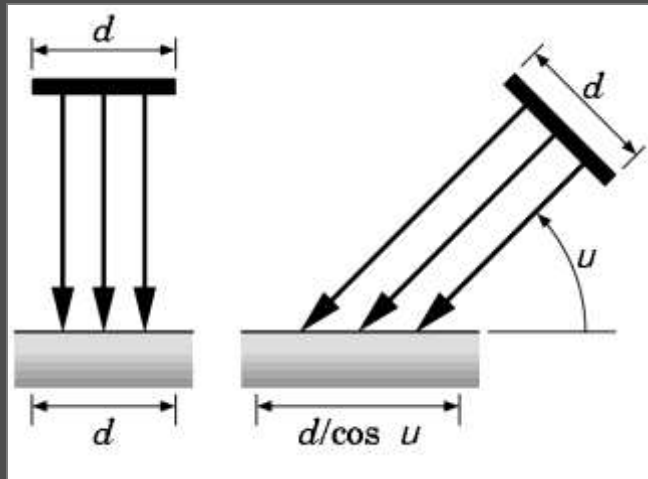
Angel (2000)

Light Interaction with Surfaces

- diffuse reflection modeled according to Lambert's law by cosine function:

$$I \sim \cos \theta = L \cdot N \text{ for normalized } L, N$$

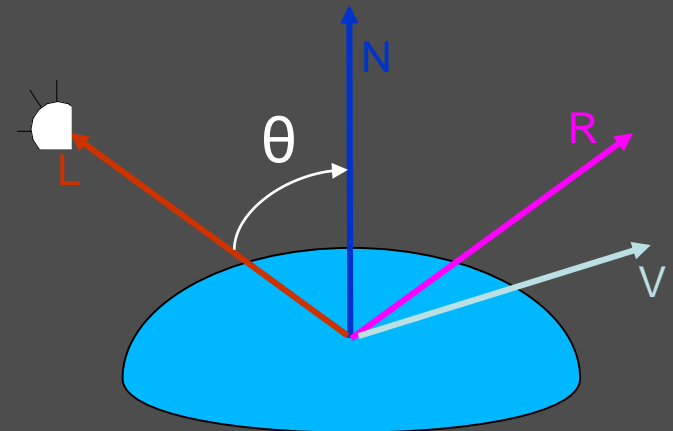
because light distributes over larger area



Angel (2000)

$\theta = 0^\circ \rightarrow \text{max. intensity}$

$\theta = 90^\circ \rightarrow 0 \text{ intensity}$

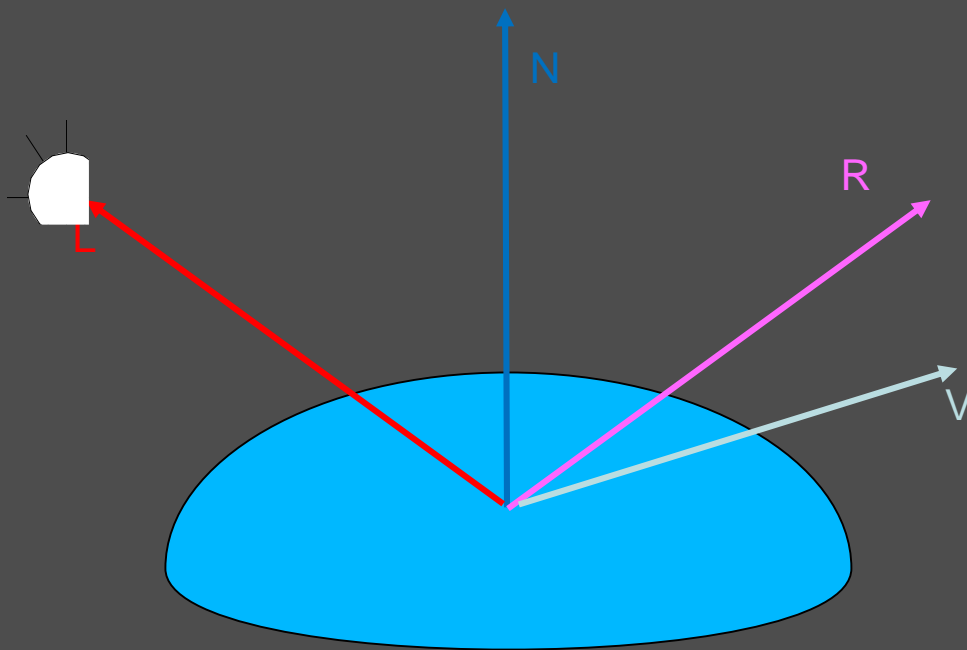


Phong Illumination Model (1973)

- most common CG model for illumination (by Bùi Tường Phong): $I_{\text{Phong}} = I_a + I_d + I_s$
- ambient light: base illumination of scene
 - simulates light scattering on objects
 - necessary because repeated diffuse reflection is not considered in local illumination model
 - depends on color of all objects in scene
 - should always be kept very small
- diffuse light: light from diffuse reflection
- specular light: light from directed reflection

Phong Illumination Model

$$I_{Phong} = I k_a + \frac{1}{a + bd + cd^2} \left(I k_d (L \cdot N) + I k_s (R \cdot V)^e \right)$$



- L – vector to light source
- N – surface normal vector
- R – reflected light ray
- V – vector to viewer/observer

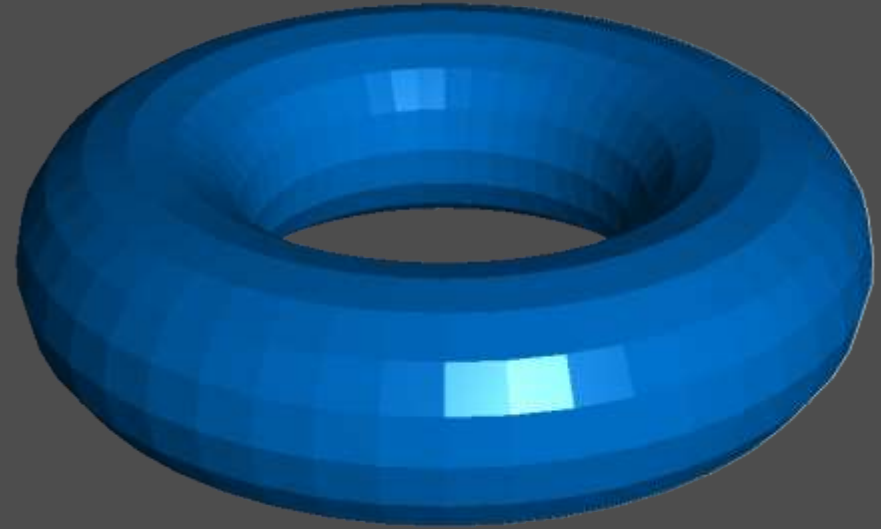
has to be evaluated for all light sources and for each of the base colors

Shading of Polygonal Models

- *status*: we can approximate color at a point
- *goal*: we want to render the whole model
- *constraint*: efficiency and quality
- *approach*: **shade** all pixels of a triangle based on color computation at a few points
- *three techniques*:
 - flat shading
 - Gouraud shading
 - Phong shading (\neq Phong illumination)
- *shading* \neq *shadow computation*

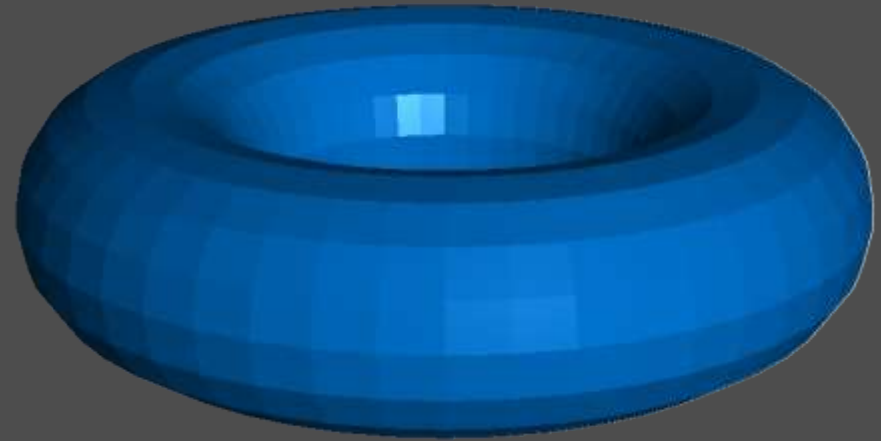
Flat Shading

- no interpolation
- all pixels same color
- two methods:
 - one point per triangle/quad
 - average of triangle's/quad's vertices
- low quality: single primitives easily visible
- fast computation & easy implementation



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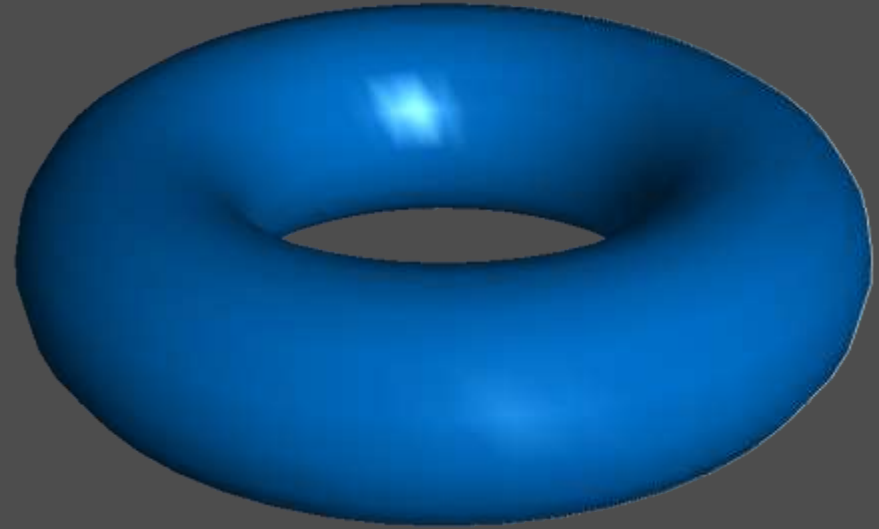
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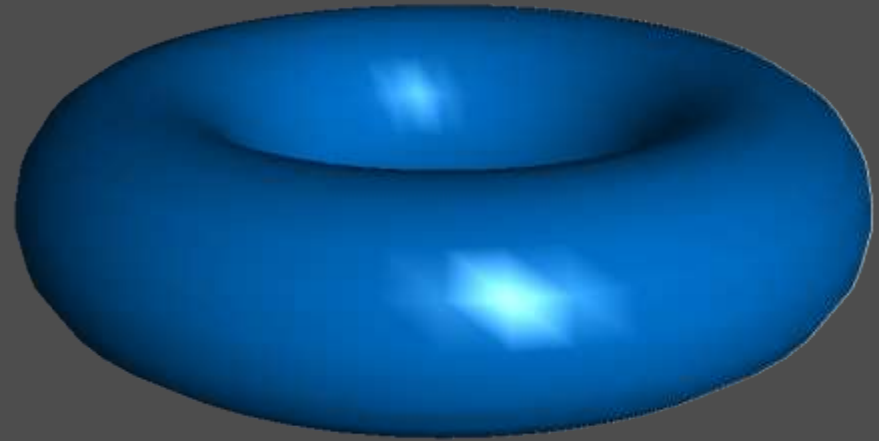
Gouraud Shading (1971)

- computation of colors at all vertices
- linear interpolation of colors over primitive
- more computation but better quality than flat shading
- usually implemented in graphics hardware
- highlights problematic: highlight shapes and highlights in the middle of triangles



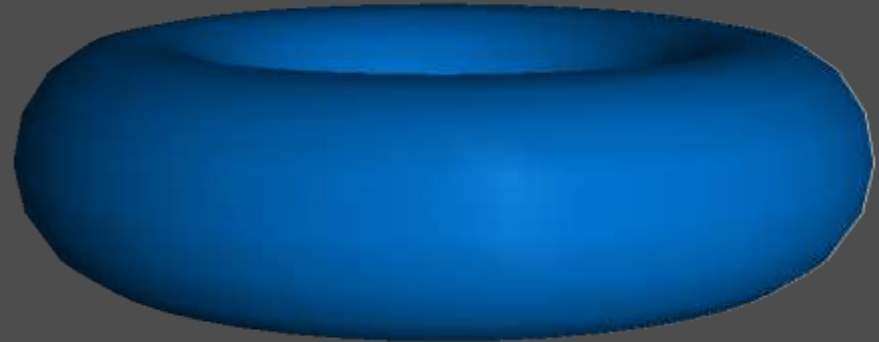
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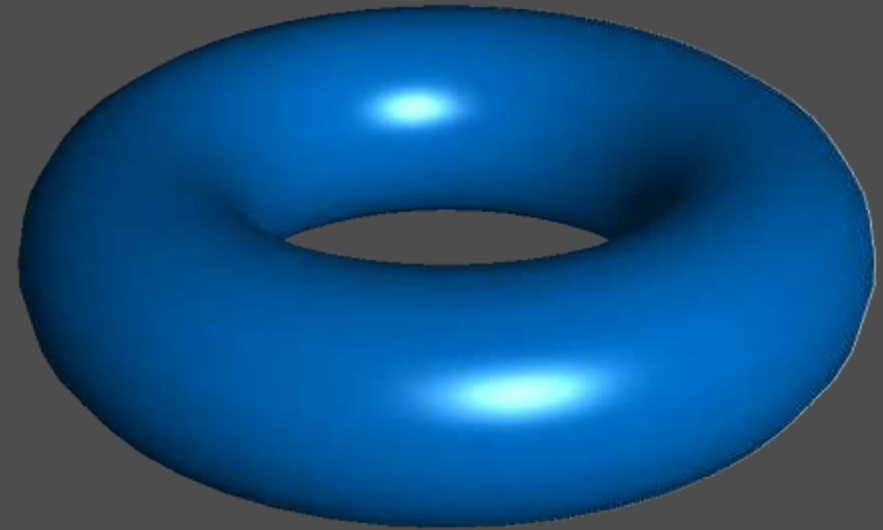
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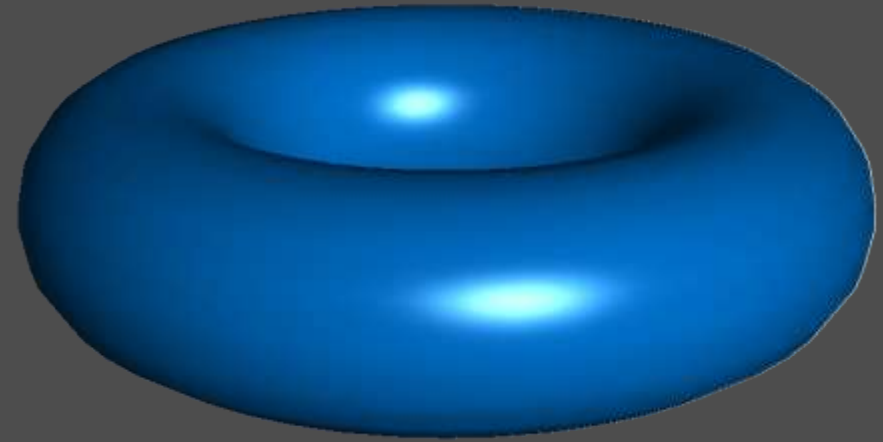
Phong Shading (1973)

- linear interpolation of normals for each pixel
- color computation for each pixel separately
- best quality, highlights are shown correctly
- but **computationally more expensive**
- problems:
 - polygons still visible at silhouettes
 - traditionally not implemented in hardware (nowadays not a problem with shaders)



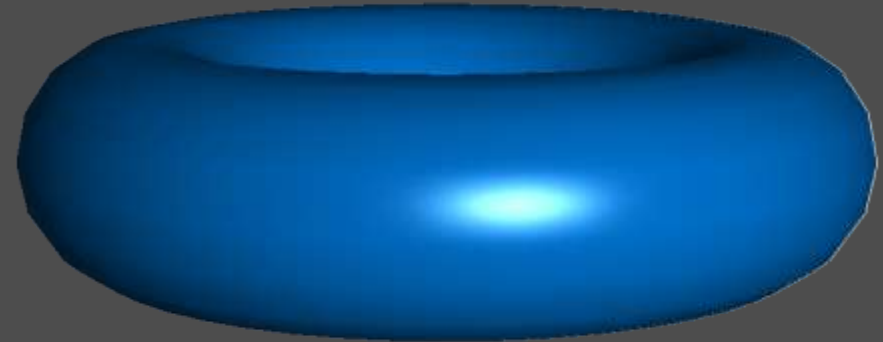
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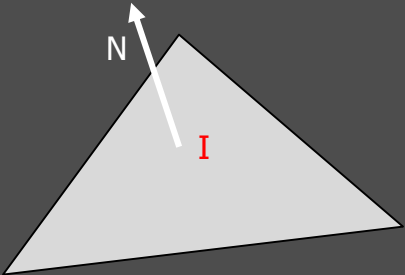
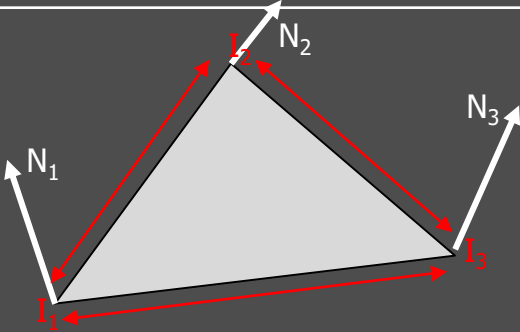
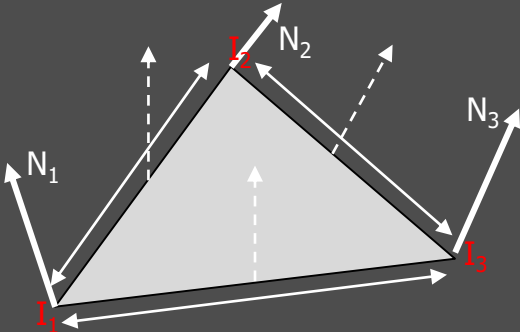
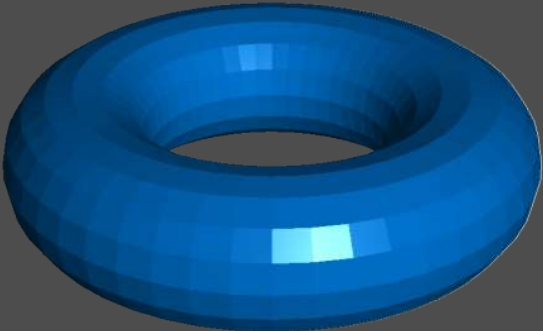
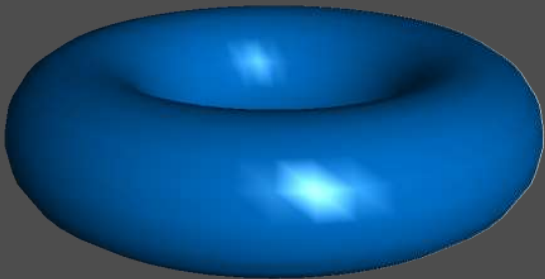
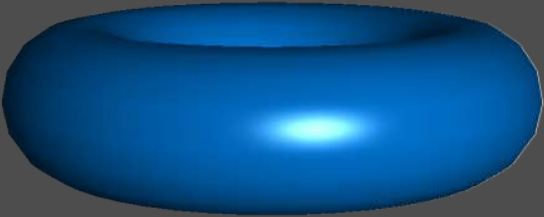


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Polygonal Shading: Comparison

Flat	Gouraud	Phong
		
one color value for entire polygon	one color value per vertex & interpolation inside the triangle	vertex normals interpolated and one color value per pixel
		

Texture Mapping



Texture Mapping Motivation

- so far: detail through polygons & materials
- example: brick wall
- problem: many polygons & materials needed for detailed structures
 - inefficient for memory and processing
- new approach necessary: texture mapping
- introduced by Ed Catmull (1974), extended by Jim Blinn (1976)

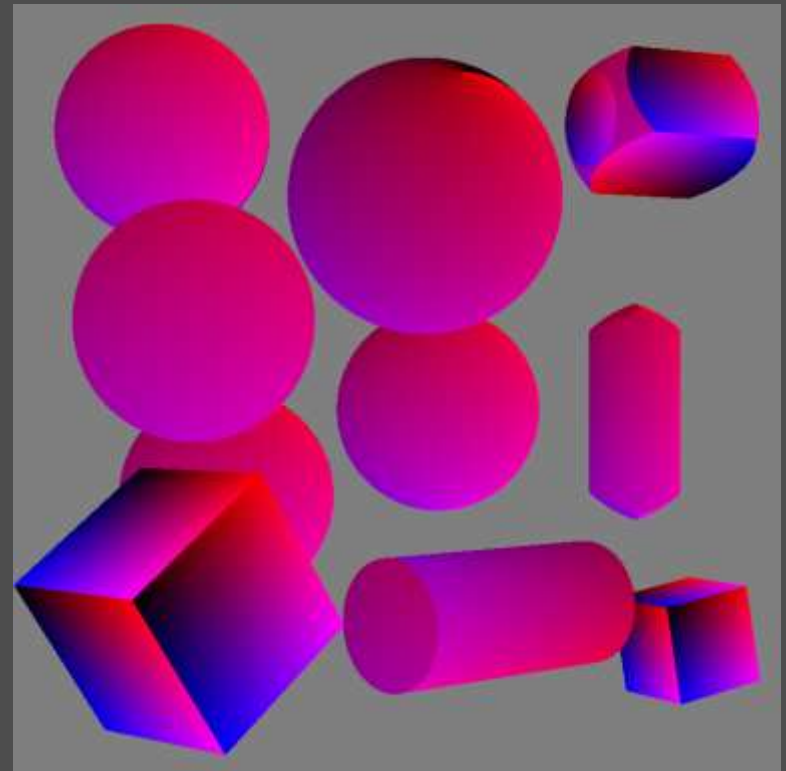


Texture Mapping Motivation

- several properties can be modified
 - color: diffuse component of surface
 - reflection: specular component of surface to simulate reflection (environment mapping)
 - normal vector: simulate 3D surface structure (bump mapping)
 - actual surface: raise/lower points to actually modify surface (displacement mapping)
 - transparency: make parts of a surface entirely or to a certain degree transparent

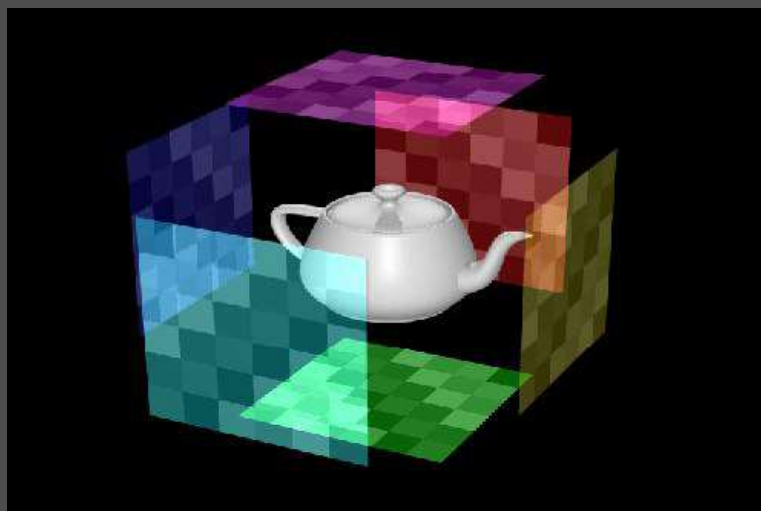
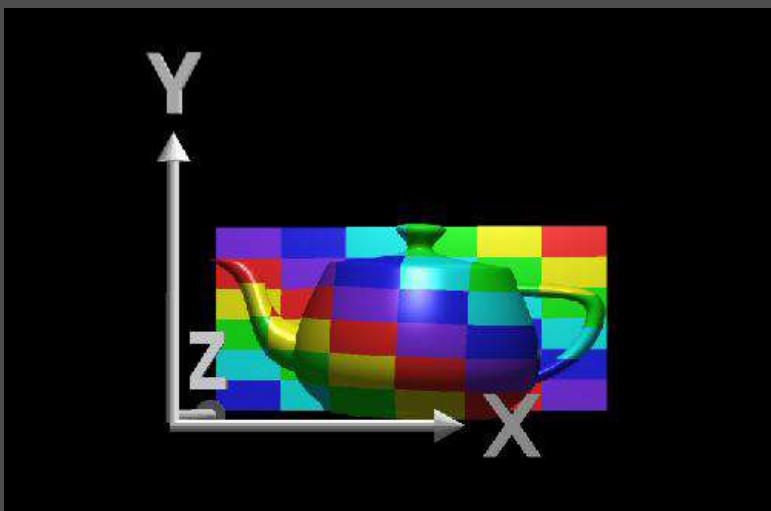
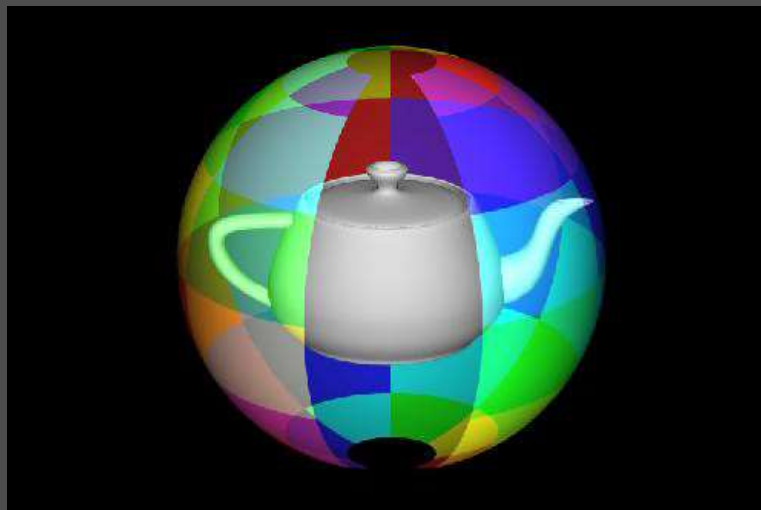
Inherent Texture Coordinates

- (u, v) coordinates derived from parameter directions of surface patches (e.g., Bézier and spline patches)
- obvious (u, v) coordinates derived for primitive shapes (e.g., boxes, spheres, cones, cylinders, etc.)



2-Step Mapping

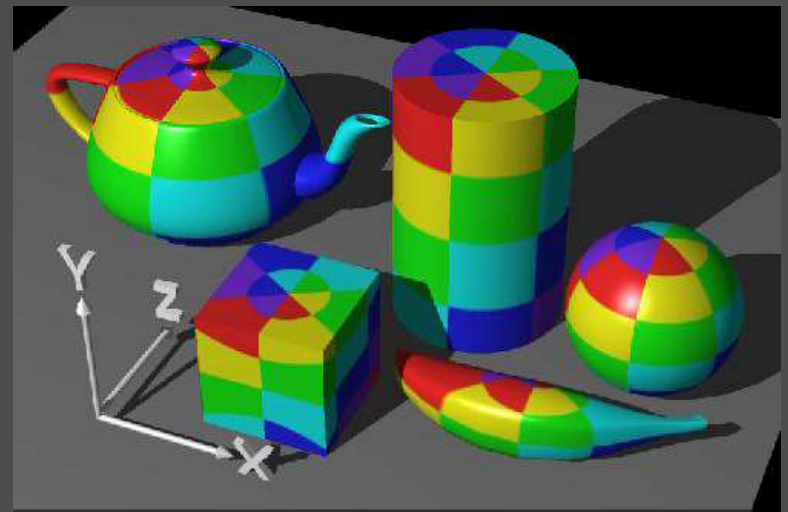
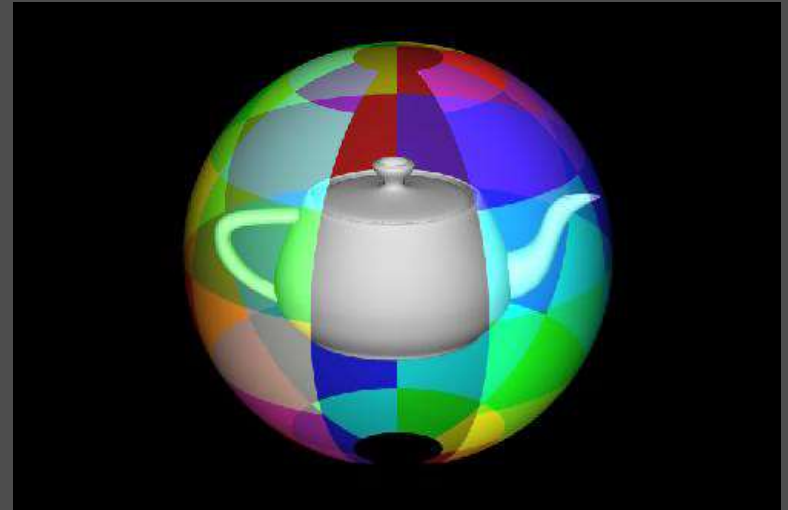
- four models: cylinder, sphere, plane, cube



from R. Wolfe: Teaching Texture Mapping

2-Step Spherical Mapping

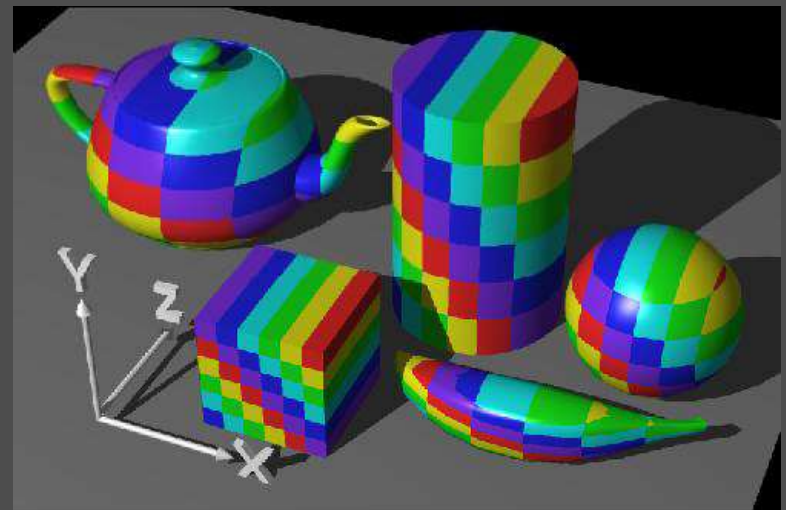
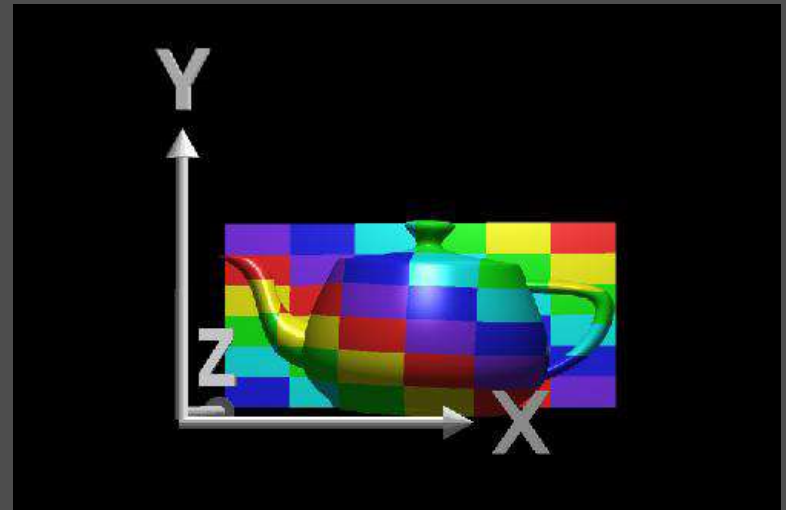
- mapping onto surface of a sphere given by spherical coordinates
- no non-distorting mapping possible between plane and sphere surface



from R. Wolfe: *Teaching Texture Mapping*

2-Step Planar Mapping

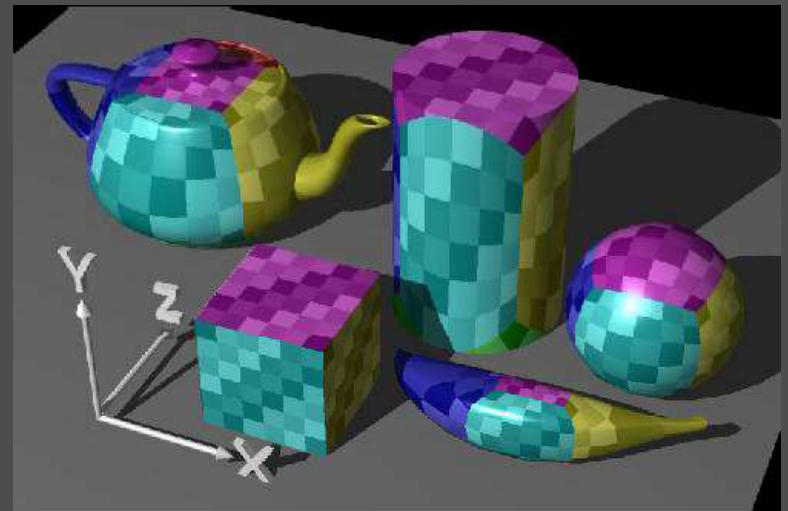
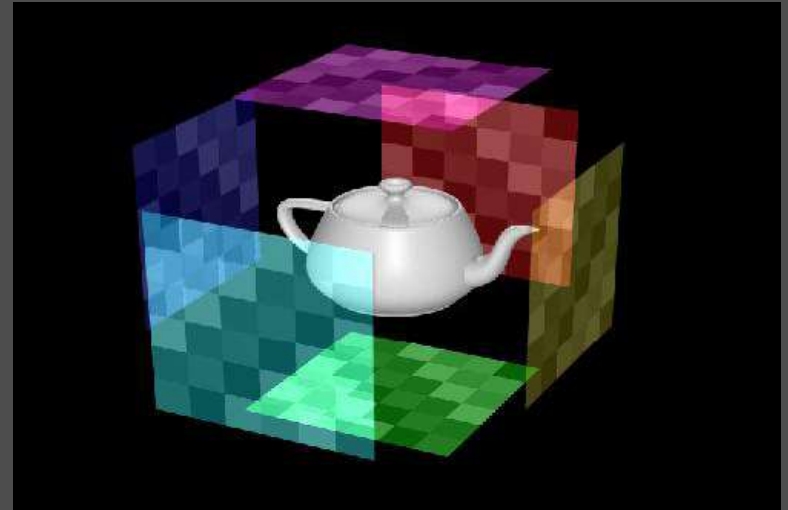
- mapping onto planar surface given by position vector and two additional vector



from R. Wolfe: *Teaching Texture Mapping*

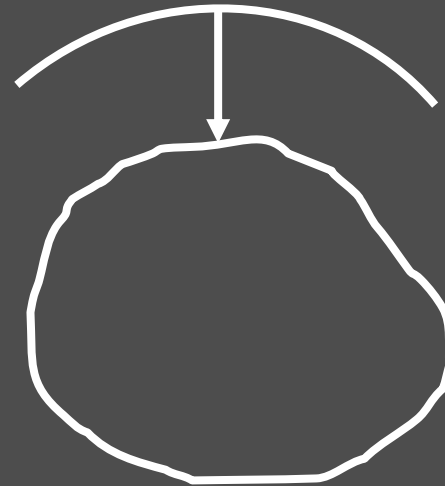
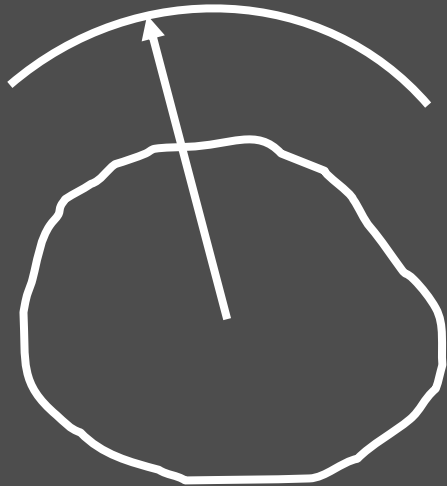
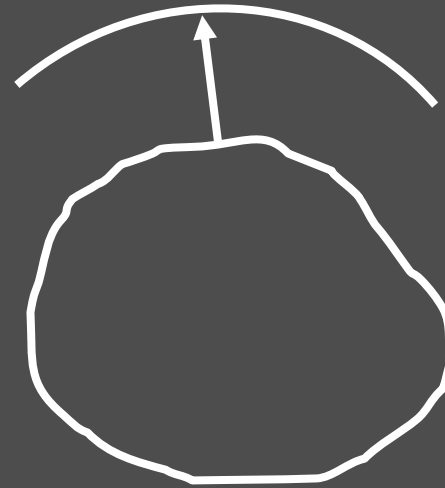
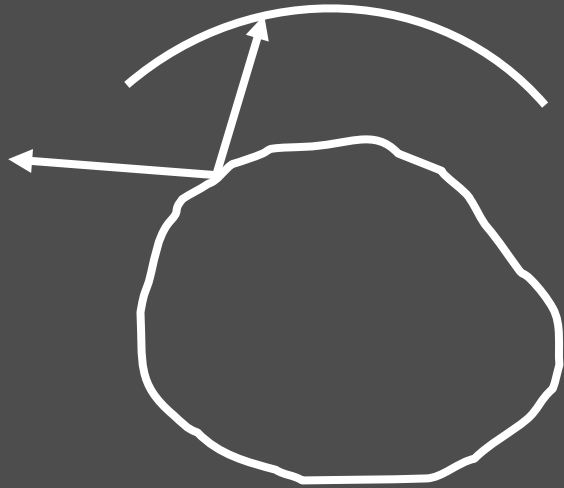
2-Step Box Mapping

- enclosing box is usually axis-parallel bounding box of object
- six rectangles onto which the texture is mapped
- similar to planar mapping



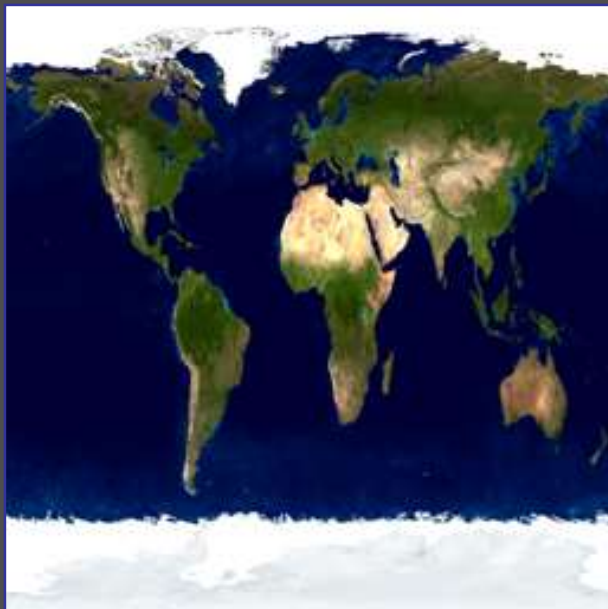
from R. Wolfe: *Teaching Texture Mapping*

O Mapping: Object to Surface



Texture Mapping: Mip Mapping

- optimal texture mapping (speed & quality): texel size \approx pixel size
- idea: use stack of textures and select the most appropriate one w.r.t. situation



256 × 256



128 × 128



64 × 64



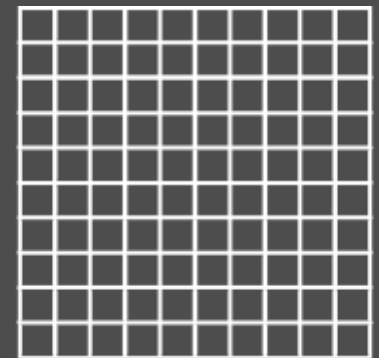
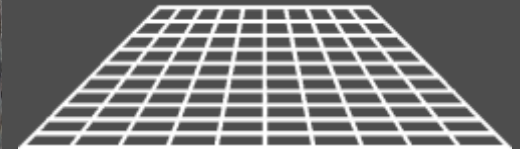
...



1 × 1

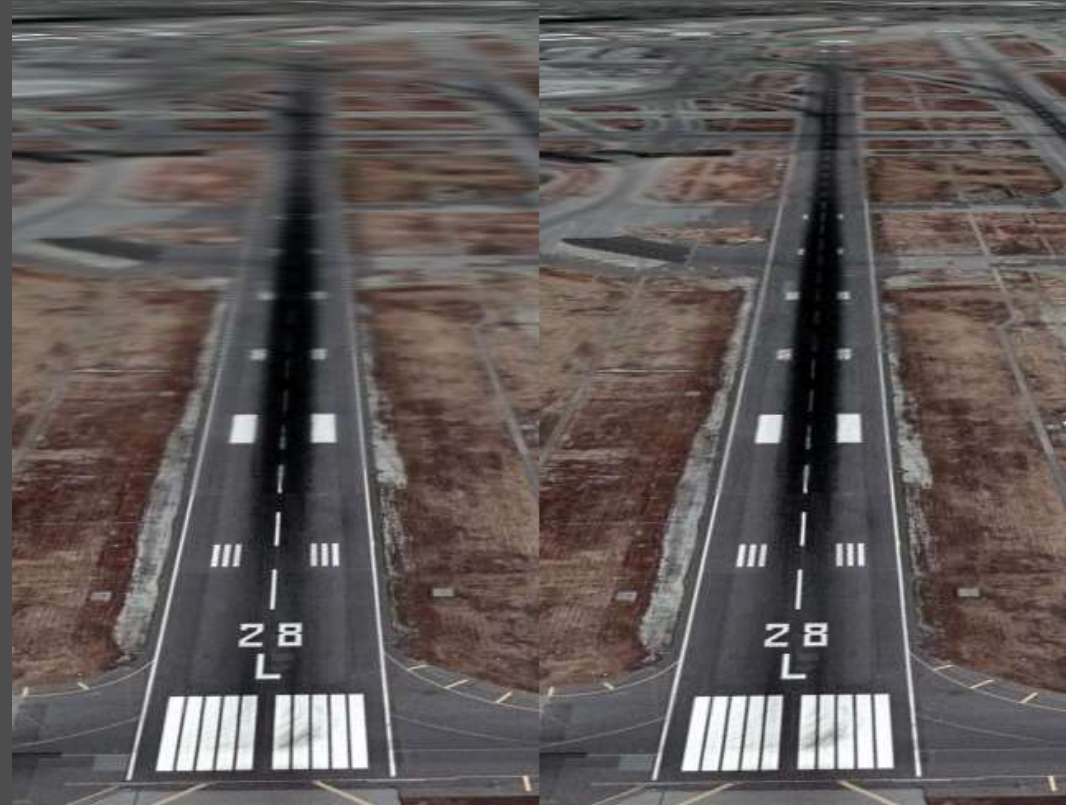
Texture Mapping: Anisotropic Filtering

- large textures not perpendicular to viewing direction: blurring problems w.r.t. angle
- appropriate mip map selection not possible

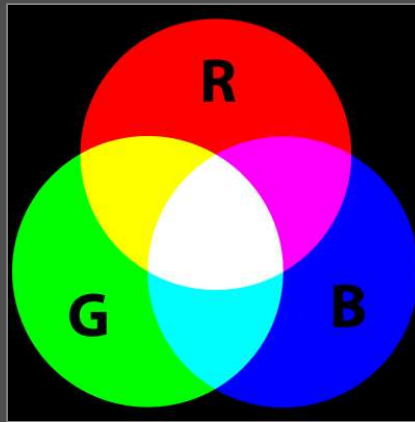


Texture Mapping: Anisotropic Filtering

- large textures not perpendicular to viewing direction: blurring problems w.r.t. angle
- appropriate mip map selection not possible
- generate mip maps favoring one direction: 256×128 , 256×64 , 128×64 , 128×32 , ...



Color and Color Models



What is color?



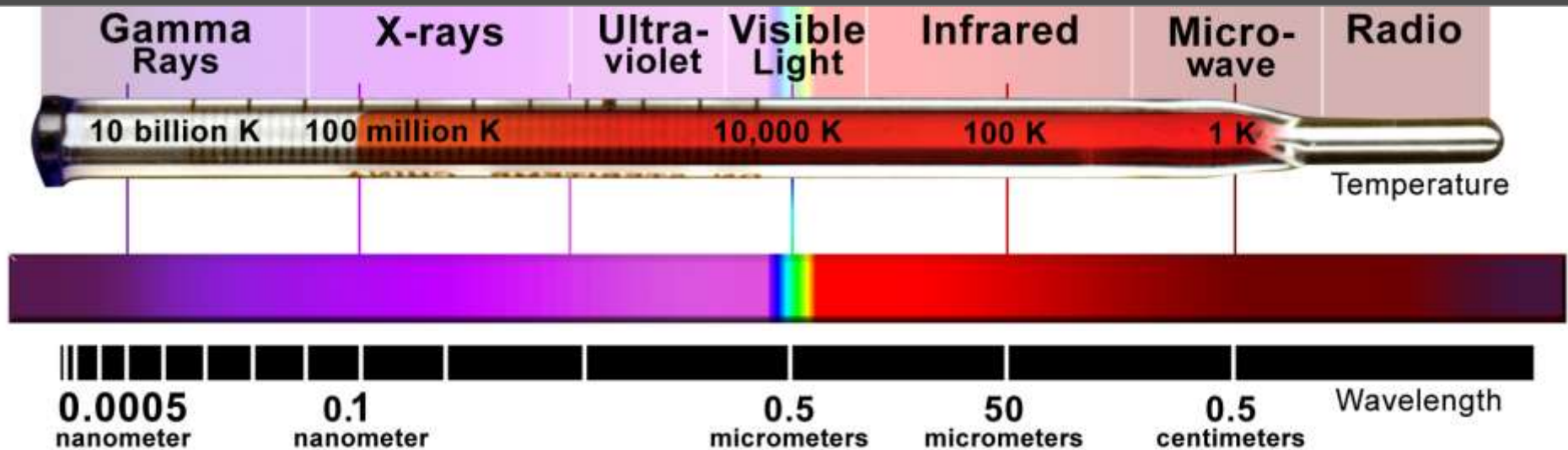
What is color?



What is Color?

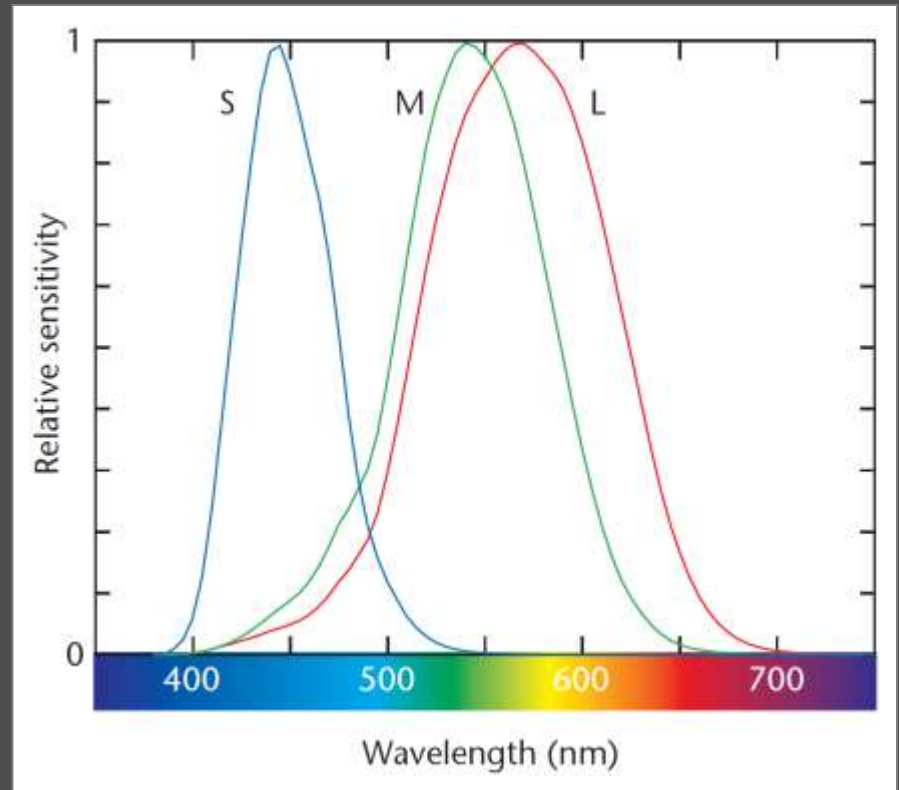
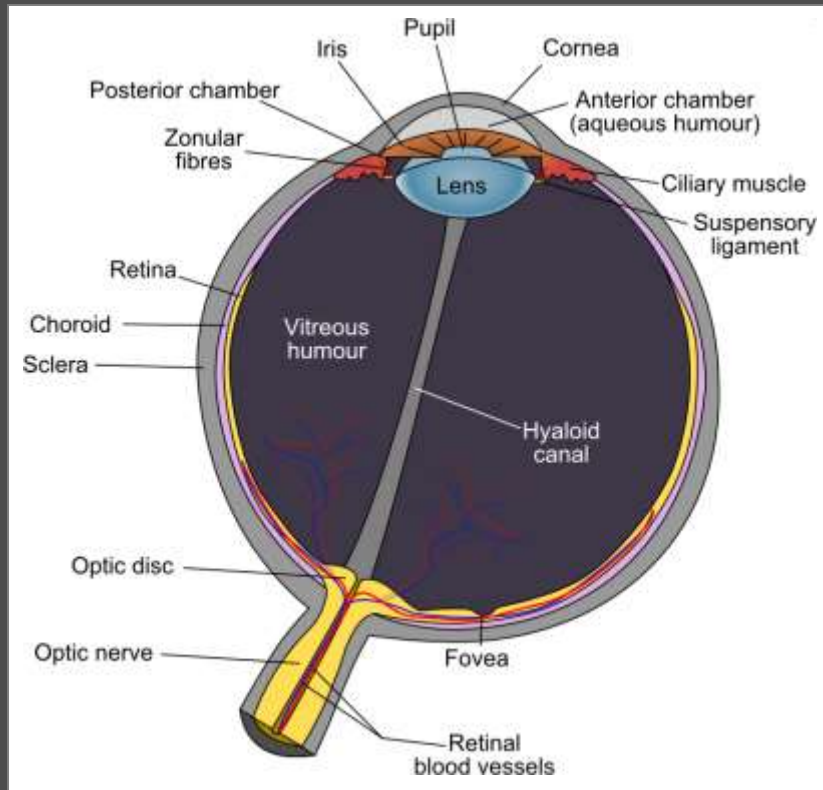
- color is a **human reaction** to light (change) (can also be influenced by cultural background)
- what is light?
- light is the visible part (370–730nm) of the electromagnetic spectrum

from NASA



Human Color Perception

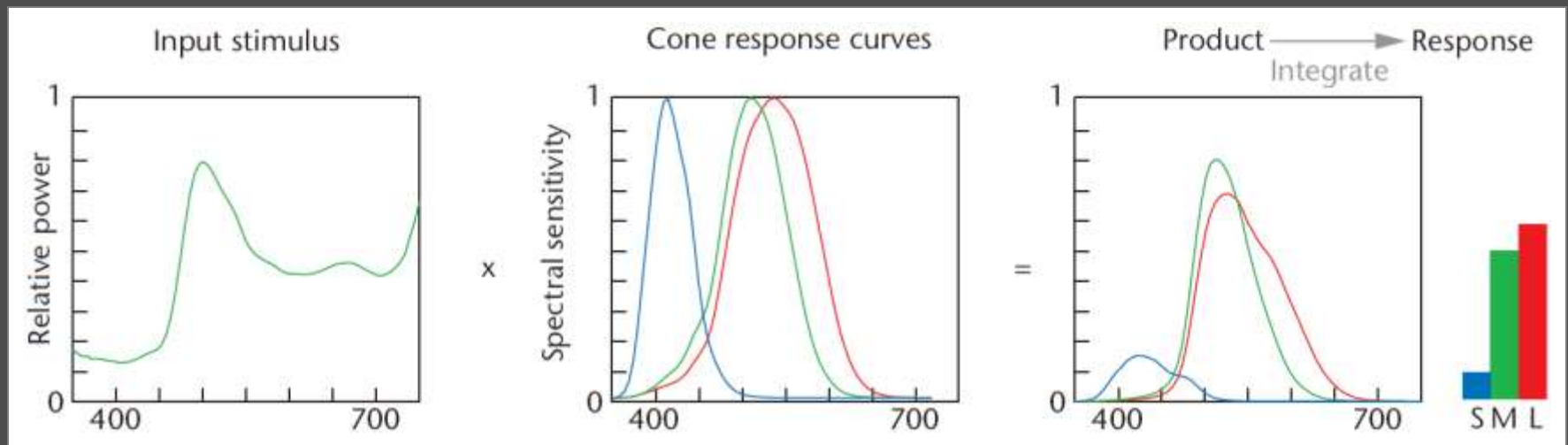
- light is converted into signals by cone cells
- three cone types with different sensitivities



Stone 2005

Human Color Perception

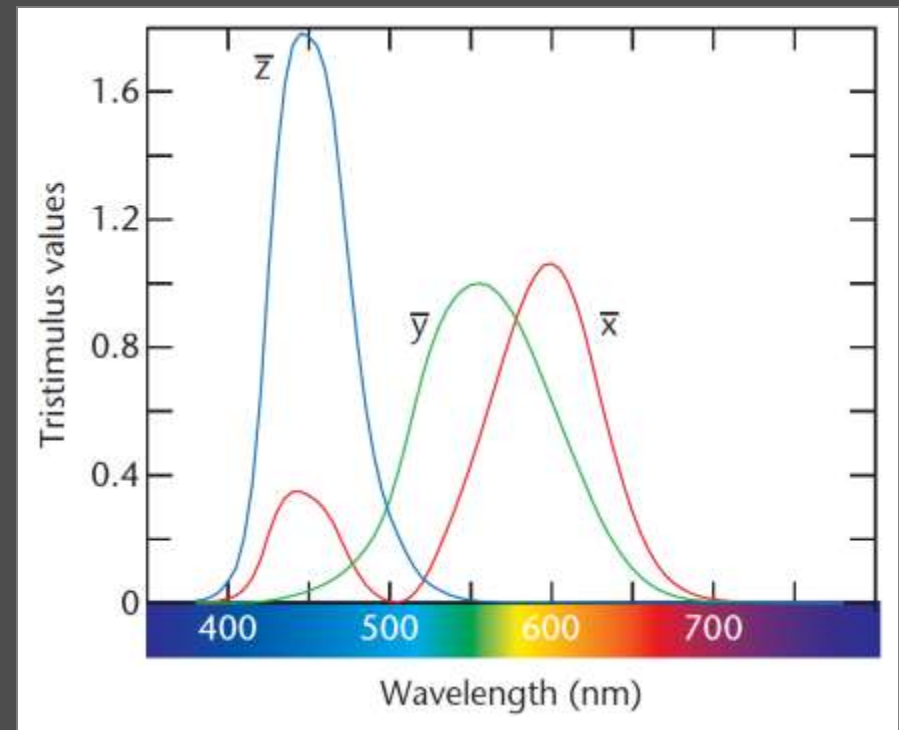
- colored light = spectral distribution function: light intensity as function of wavelength
- converted into 3 response values by cones (short, medium, and long wavelengths)



Stone 2005

Describing Color Vision: XYZ Color Model

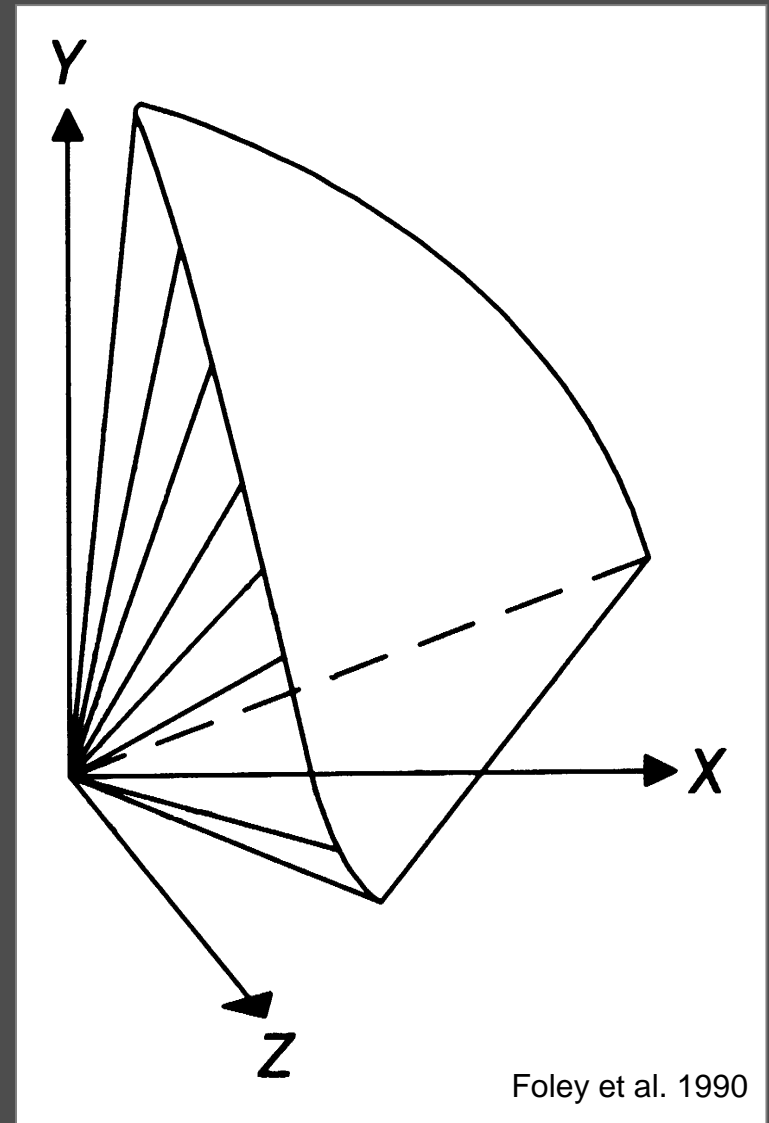
- definition of three primary colors: X, Y, Z
 - color-matching functions are non-negative
 - Y follows the standard human response to luminance, i.e., the Y value represents perceived brightness
 - can represent all perceivable colors
- mathematically derived from experiments



Stone 2005

XYZ CIE Color Space

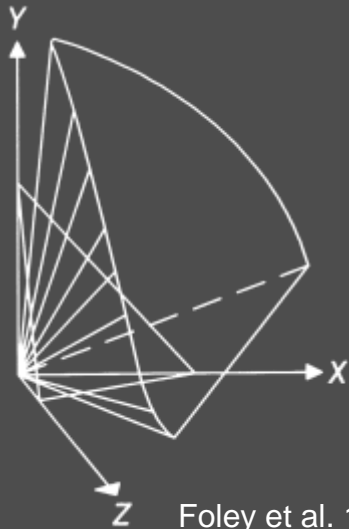
- plotting XYZ space in 3D
- all colors that are perceivable by humans form a deformed cone
- X , Y , and Z -axes are outside this cone



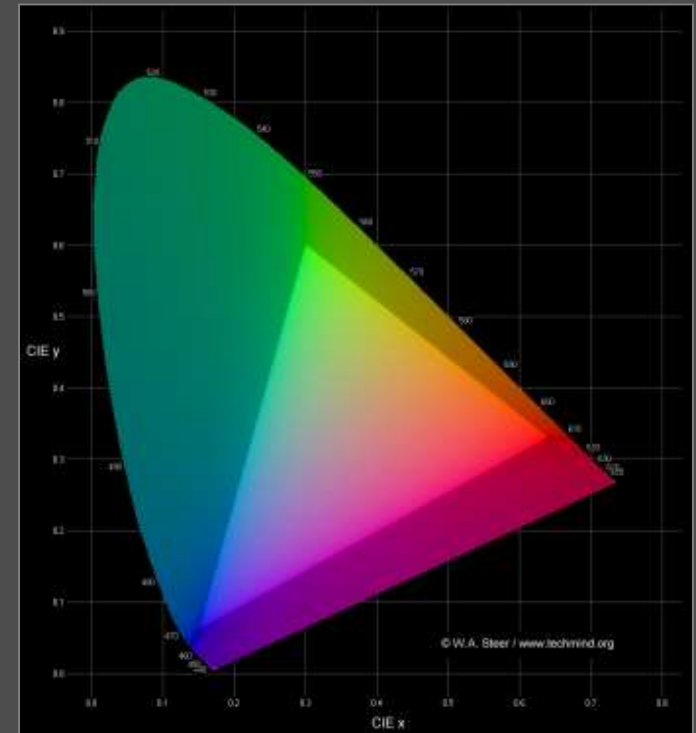
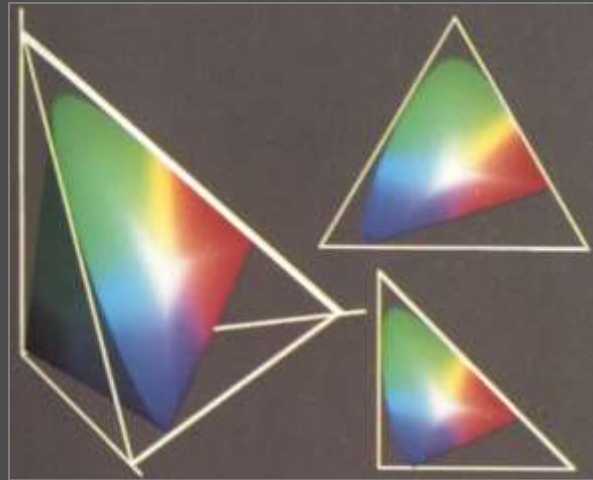
CIE Chromaticity Diagram

- projection of XYZ space onto $X+Y+Z = 1$ (to factor out a color's brightness):
 $x = X/(X+Y+Z)$ $y = Y/(X+Y+Z)$
- monochromatic colors on upper edge
- color gamut: colors visible on a device through color adding

<http://www.techmind.org/>

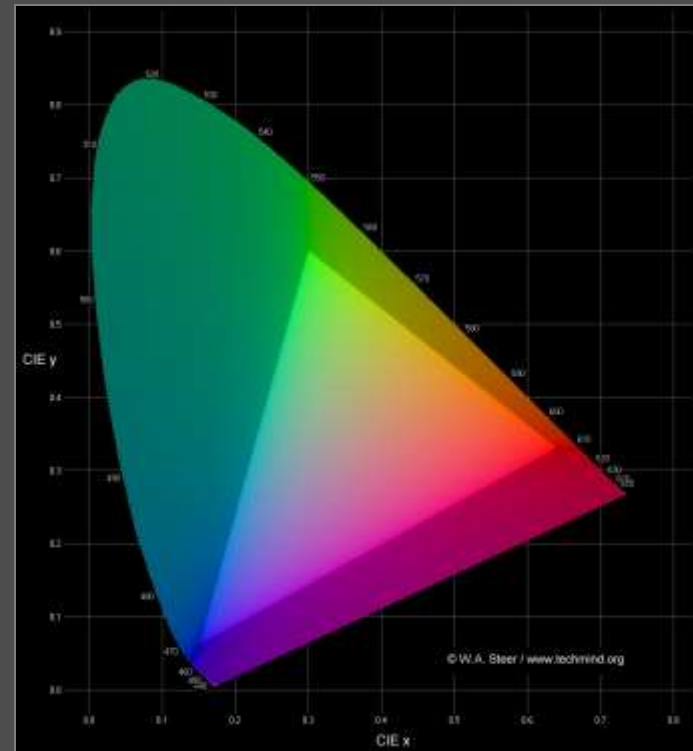
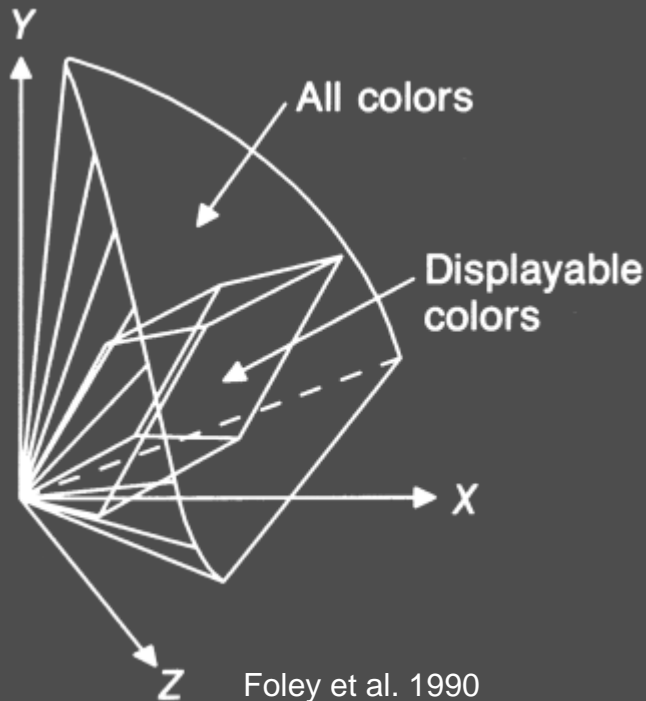


Foley et al. 1990



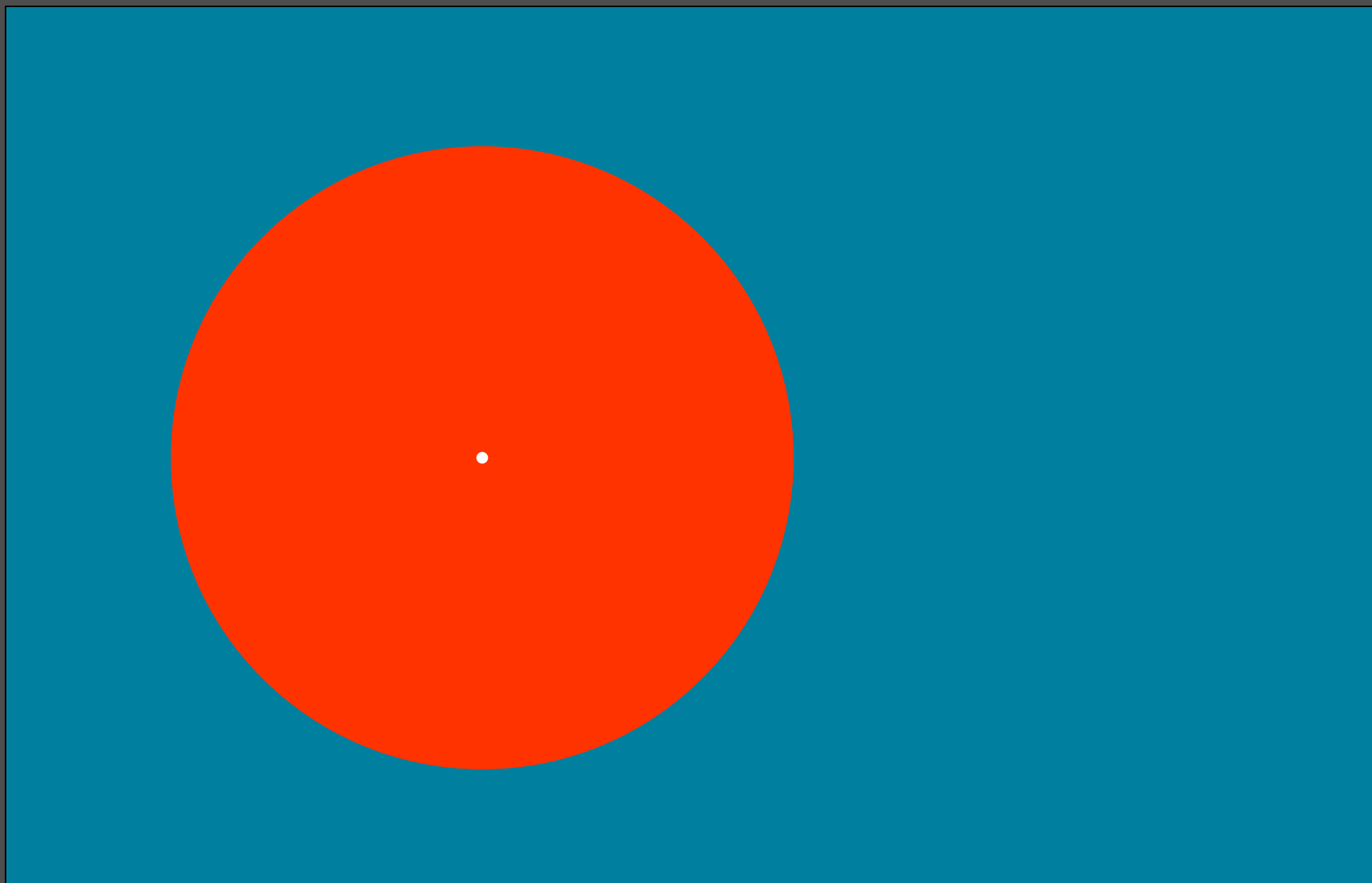
Can RGB Represent Any Color?

- no, because all colors form horseshoe shape in CIE chromaticity diagram and RGB gamut is triangular

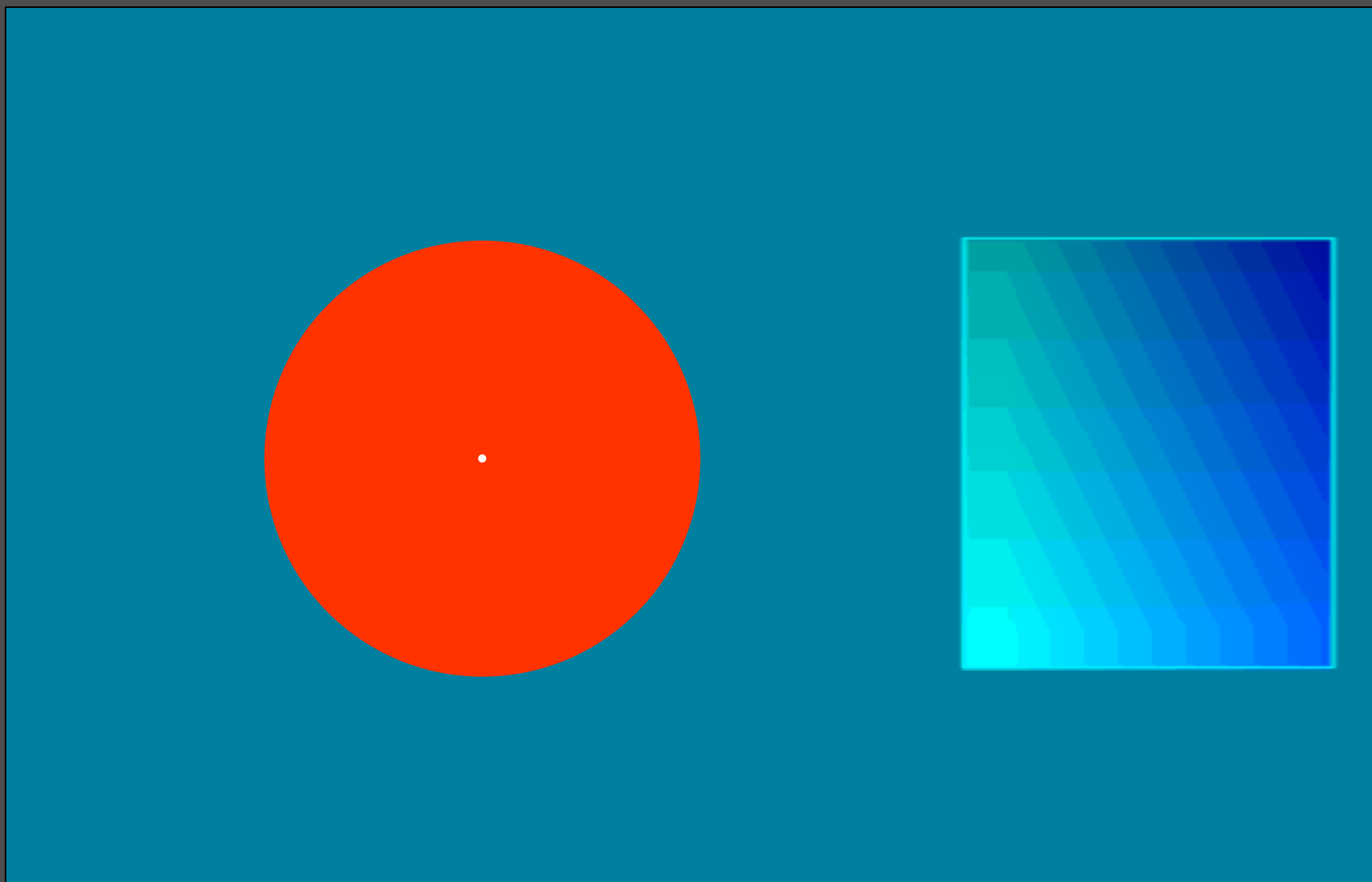


<http://www.techmind.org/>

Let's see REAL cyan ...

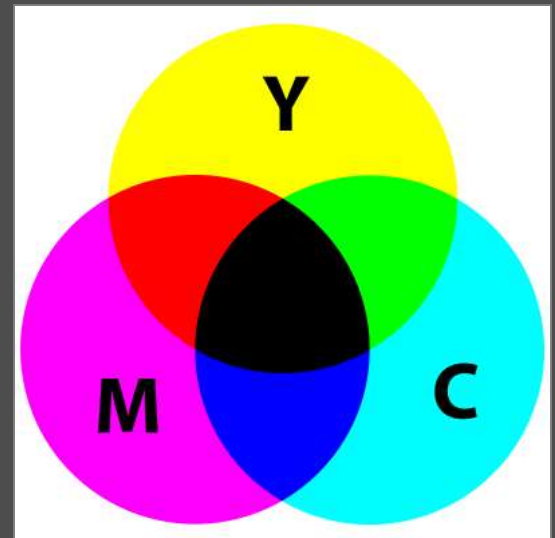
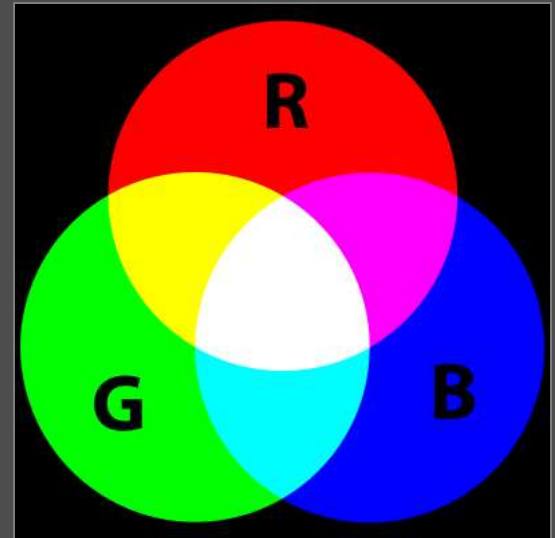


Let's see REAL cyan ...



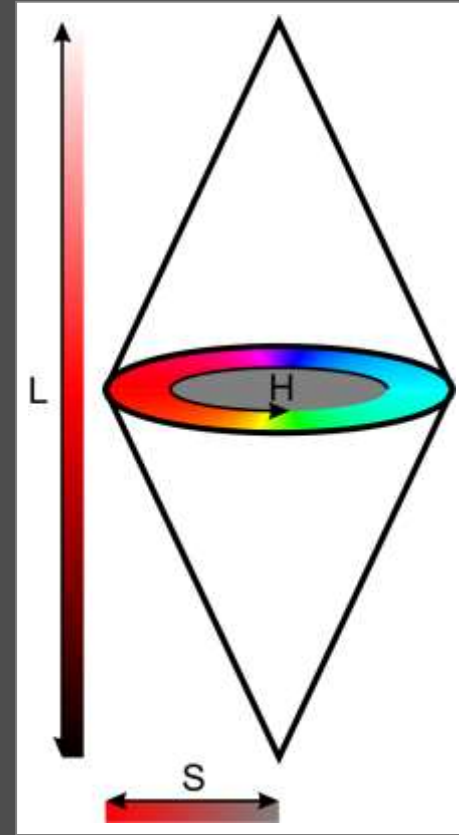
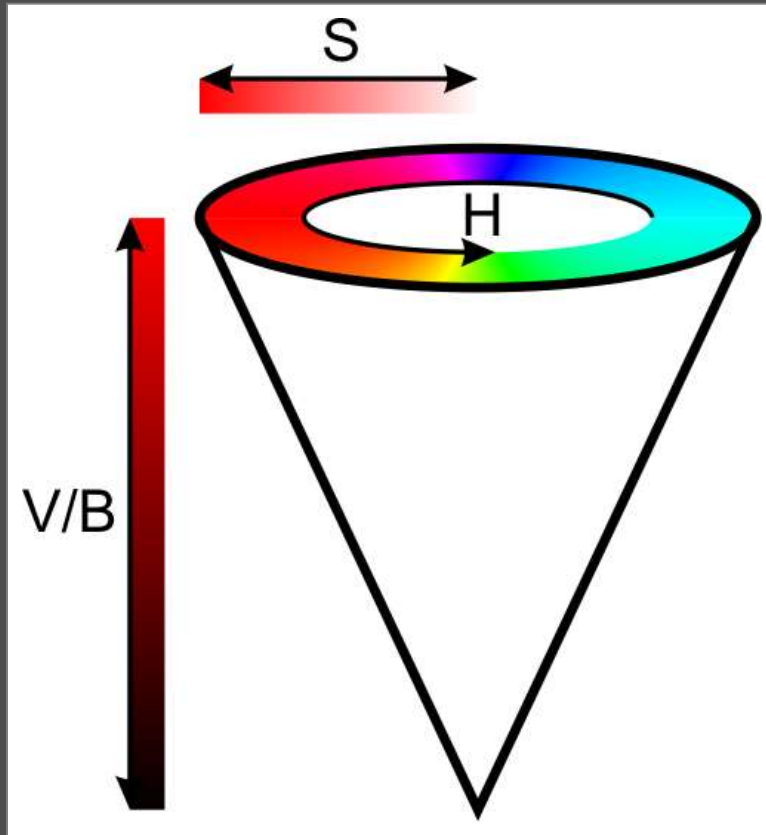
Additive vs. Subtractive Color

- (physical) color mixing depends on color production process
- device-dependent color models
 - light emission:
additive mixing
(CRTs etc.): **RGB model**
 - light absorption:
subtractive mixing
(printing process): **CMY(K) model**



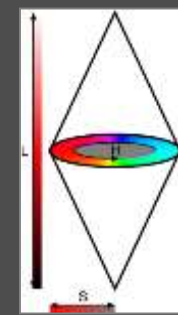
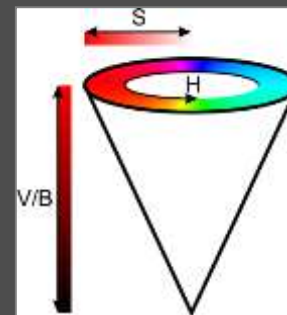
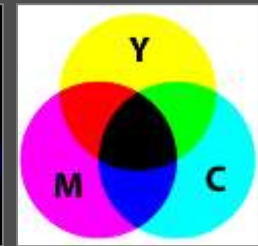
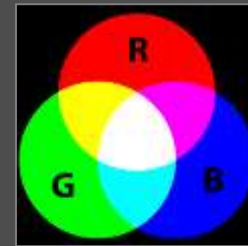
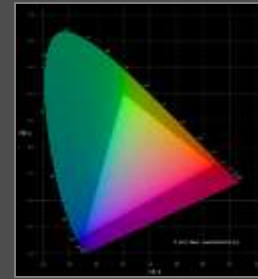
HSV/HSB and HSL/HLS Color Models

- human's inefficient with device models
- perceptual models to ease color specification



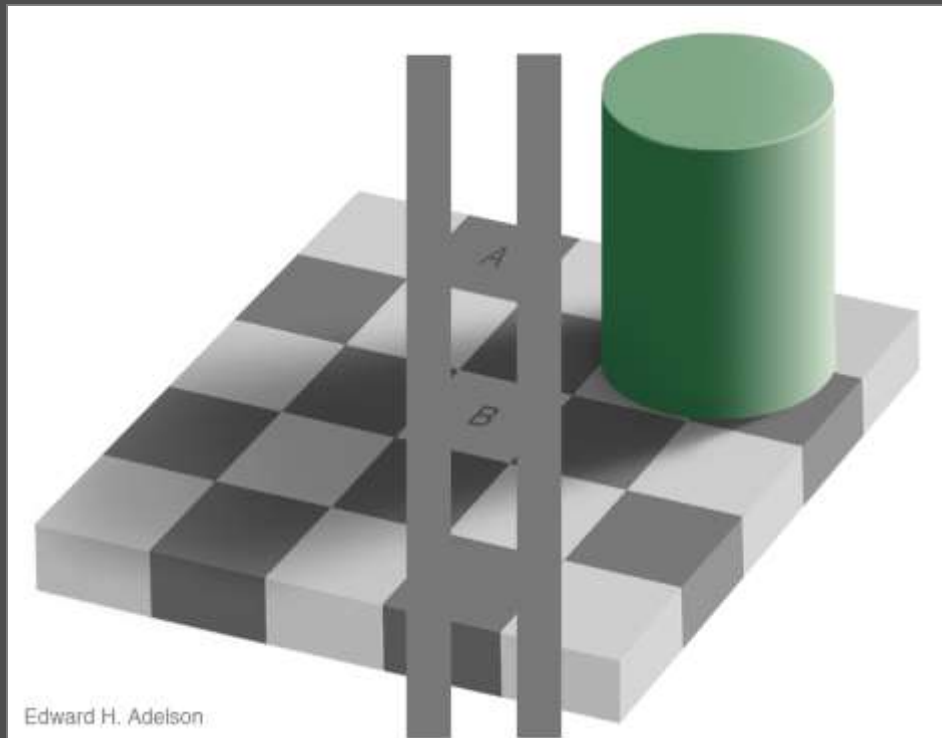
Color Models Summary

- human vision-oriented
 - CIE XYZ & LMS
- hardware-oriented
 - RGB & CMY(K)
- perceptual models
 - HSV/HSB & HSL/HLS
- other models (e.g., for TV)
 - YIQ (NTSC) & YUV (PAL)



Other Color Topics

- color perception depending on context
- color deficiency



Other Color Topics

- color perception depending on context
- color deficiency

